THE BASIC EM DIFFERENTIAL EQUATIONS

$\nabla \times \tilde{E} = -j\omega\mu \tilde{H} \xrightarrow{\nabla \times} \nabla^2 \tilde{E} + k^2 \tilde{E} = j\omega\mu \tilde{J} + \frac{1}{\varepsilon}\nabla\rho$

Similarly: $\nabla^2 \tilde{H} + k^2 \tilde{H} = -\nabla \times \tilde{J}$

} Inhomogeneous Helmholtz Equations

contain only 1 field in each equation.

where: $k^2 = \omega^2 \mu\varepsilon$

When there are no sources:

$$\left.\begin{aligned} \nabla^2 \tilde{E} + k^2 \tilde{E} = 0 \\ \nabla^2 \tilde{H} + k^2 \tilde{H} = 0 \end{aligned}\right\} \text{Homogeneous}$$

(eg. $\nabla^2 E_x + k^2 E_x = 0$)

For quasistatic (or static) problems ("steady-state") $k^2 \to 0$

$$\left.\begin{aligned} \nabla^2 \tilde{E} = 0 \\ \nabla^2 \tilde{H} = 0 \end{aligned}\right\} \begin{array}{l}\text{Laplace equations} \\ \text{(separate into coordinate components)}\end{array}$$

For this type of problems it is convenient to use the scalar potential equations

$$\tilde{E} = -\nabla\Phi, \quad \tilde{H} = -\nabla\Phi_m$$

with Φ, Φ_m satisfying Laplace equations.

$$\nabla^2\Phi = 0, \quad \nabla^2\Phi_m = 0 \quad \text{scalar eq.}$$

For regions containing charges, the Poisson equation applies to Φ.

$$\nabla^2\Phi = -\frac{\rho}{\varepsilon}$$

Unique solutions of Laplace / Poisson equation result if the function or its derivative is specified on a boundary surrounding the region of interest. Unique solutions of Helmholtz equation are obtained by specifying the tangential component of $\tilde{E}$ or $\tilde{H}$ on the closed boundary or tangential $\tilde{E}$ on a part of the boundary and tangential $\tilde{H}$ on the remainder.

Superposition

∇^2 is a linear operator

Any two solutions are superposable and their sum is a solution provided that the medium itself is linear.

Separation of Variables

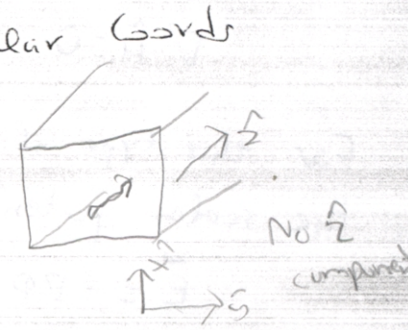
Solutions that are products of 2 (3) function in 2D (3D) each function depending upon one coordinate variable only.

$\downarrow$

can be added to form a series.

Single-product solutions of the wave equation represent modes which can propagate individually (Important in waveguides and resonant structures).

Assume: 2D Laplace equation in Rectangular Coords



$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0$$

Solutions: $\varphi(x,y) = X(x)\, Y(y)$ separation of vars

$$\Downarrow$$

$$X''Y + XY'' = 0 \Rightarrow \frac{X''}{X} + \frac{Y''}{Y} = 0 \qquad \forall x, y$$

Since the second term doesn't contain $x \Rightarrow$ cannot vary with x

$\Rightarrow$ the first term cannot vary with x either $\Rightarrow$ constant

Set: $\frac{X''}{X} = k_x^2$, $\frac{Y''}{Y} = k_y^2 \Rightarrow \begin{cases} k_x^2 + k_y^2 = 0 \\ X'' - k_x^2 X = 0 \\ Y'' - k_y^2 Y = 0 \end{cases}$ $k_x^2 = -k_y^2 \rightarrow k_x, k_y \in \mathbb{I}$ (or)

$\downarrow$

$k_x = k$, $k_y = \pm jk$

or

$k_x = \pm jk$, $k_y = k$

$X(x) = Ae^{k_x x} + Be^{-k_x x}$

$Ae^{c_1 x} + Be^{c_2 x} \rightarrow c^2 - k_x^2 = 0 \rightarrow c = \pm k_x$

$Y(y) = Ce^{k_y y} + De^{-k_y y}$

$\varphi(x,y) = (A \cosh kx + B \sinh kx)(C' \cos ky + D' \sin ky)$

$= XY = (Ae^{k_x x} + Be^{-k_x x})(Ce^{jk_y y} + De^{-jk_y y})$

$\varphi(x,y) = (A \cos kx + B \sin kx)(C \cosh ky + D \sinh ky)$

The choice depends on Boundary conditions

If $\exists$ repeated zeros $\to y$ use $(C\cos ky + D\sin ky)$

If $\exists$ " $\to x$ use $(A\cos kx + B\sin kx)$

If boundary $\to$ infinity, use real exponentials instead of sinh, cosh

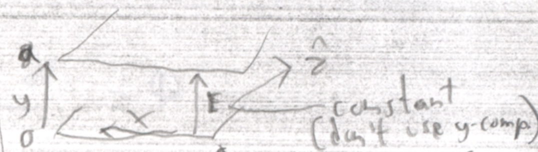
If $k_x = jk_y = 0 \Rightarrow \varphi(x,y) = (A_1 x + B_1)(C_1 y + D_1)$ linear dependence (TE, TM) No change in x,y dir.

$\rightsquigarrow \hat{x} \quad e^{-k_x x}$

$\rightsquigarrow -\hat{x} \quad e^{k_x x}$

$\cos x = \dfrac{e^{jx} + e^{-jx}}{2}$

$\cosh x = \dfrac{e^{x} + e^{-x}}{2}$



$E_t = 0 \Big|_{y=a}^{y=0}$ $\sim \cos, \sin (y)$ $\sim \exp(x)$

3D – Laplace

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0$$

$$\varphi(x,y,z) = X(x)\,Y(y)\,Z(z)$$

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = 0$$

$$k_x^2 + k_y^2 + k_z^2 = 0$$

$$\varphi(x,y,z) = [A\cosh k_x x + B\sinh k_x x]\cdot[C\cosh k_y y + D\sinh k_y y]\cdot[E\cosh k_z z + F\sinh k_z z]$$

Single Rectangular Harmonic

$A = C = 0$

$\varphi = C_1 \sin kx \ \sinh ky$

$\varphi(y=0) = 0 \quad \forall x$

$\Downarrow$

one boundary can be zero-potential plane at $y=0$

$\varphi(x=0) = 0 \quad \forall y$

and at planes: $kx = n\pi$

Assume: $0 < kx < \pi$, $0 < y < \infty$. Let $ka = \pi$

$\hookrightarrow \varphi(ka = \pi)$. $\Rightarrow k = \dfrac{\pi}{a}$

Assume: $\varphi(x=a/2, y=b) = V_0$

$$\Downarrow$$

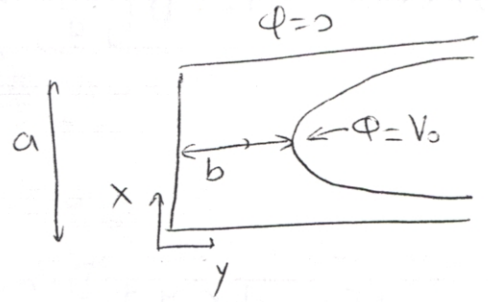
$$V_0 = C_1 \sin\frac{\pi}{2}\sinh\frac{\pi b}{a} = C_1 \sinh\frac{\pi b}{a}$$

$$\Downarrow$$

$$C_1 = \frac{V_1}{\sinh\frac{\pi b}{a}}$$

$$\Downarrow$$

$$\varphi = \frac{V_0 \sinh(\pi y/a)}{\sinh(\pi b/a)} \sin\frac{\pi x}{a}$$



FOURIER SERIES

A single-product solution could satisfy only very special B.C's. For general boundaries, a sum of such solutions is used.

Fourier series represent periodic functions

$$f(x) = f(x+L) \quad , \quad L: \text{period}$$

$$f(x) = a_0 + a_1 \cos kx + a_2 \cos 2kx + a_3 \cos 3kx + \cdots$$
$$+ b_1 \sin kx + b_2 \sin 2kx + b_3 \sin 3kx + \cdots$$

$$(kL = 2\pi)$$

Orthogonality: $\int_{-L/2}^{L/2} \cos nkx \cos mkx = 0, \; m \neq n$, $\int_{-L/2}^{L/2} \sin nkx \sin mkx = 0, \; m \neq n$

$$\int_{-L/2}^{L/2} \sin mkx \cos nkx \, dx = 0 \quad \begin{cases} m \neq n \\ m = n \end{cases}$$

$$\int_{-L/2}^{L/2} \cos^2 mkx \, dx = \int_{-L/2}^{L/2} \sin^2 mkx \, dx = L/2$$

$$a_n = \frac{2}{L}\int_{-L/2}^{L/2} f(x) \cos nkx \, dx$$

$$b_n = \frac{2}{L}\int_{-L/2}^{L/2} f(x) \sin nkx \, dx$$

$$a_0 = \frac{1}{L}\int_{-L/2}^{L/2} f(x) \, dx$$

Fourier Integral

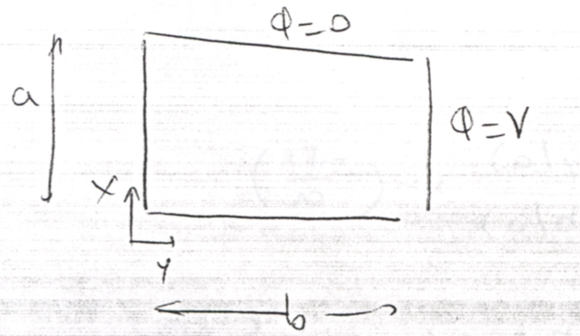
Entire domain (Aperiodic) Functions

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g(k) e^{jkx} dx$$

$$g(k) = \int_{-\infty}^{\infty} f(x) e^{-jkx} dx$$

with : $\int_{-\infty}^{\infty} |f(x)| \, dx < \infty$

2D Problem with Specified Boundary Potentials



$\varphi(x=0) = \varphi(x=a) = 0 \Rightarrow$ sinusoid to x

$\varphi(x,y) = (A\cos kx + B\sin kx)(C\cosh ky + D\sinh ky)$

$$\begin{cases} \varphi(y=0)=0 \Rightarrow C=0 \\ \varphi(x=0)=0 \Rightarrow A=0 \\ \varphi(x=a)=0 \Rightarrow ka=m\pi \Rightarrow k=\frac{m\pi}{a} \end{cases}$$

$$\Rightarrow \varphi = C_m \sin\frac{m\pi x}{a} \sinh\frac{m\pi y}{a}$$

To satisfy the BC $|_{y=b}$ $\rightsquigarrow$

$$\Phi = \sum_{m=1}^{\infty} C_m \sin\frac{m\pi x}{a}\sinh\frac{m\pi y}{a}$$

$$\Big\{\ \Phi(y=b) = V_0,\quad 0<x<a$$

$$V_0 = \sum_{m=1}^{\infty} C_m \sin\frac{m\pi x}{a}\sinh\frac{m\pi b}{a},\quad 0<x<a$$

$\Downarrow$

Fourier expansion of V_0 over $0<x<a$

$$f(x) = V_0 = \sum_{m=1}^{\infty} a_m \sin\frac{m\pi x}{a},\quad 0<x<a$$

$$a_m = \begin{cases} \dfrac{4V_0}{m\pi}, & m \text{ odd} \\ 0, & m \text{ even} \end{cases}$$

$\Downarrow$

$$C_m \sinh\frac{m\pi b}{a} = a_m$$

$\Downarrow$

$$\Phi = \sum_{m\,\text{odd}} \frac{4V_0}{m\pi}\,\frac{\sinh(m\pi y/a)}{\sinh(m\pi b/a)}\,\sin\left(\frac{m\pi x}{a}\right)$$

⚠ nonzero condition, potential or normal derivative of potential, must exist on some part of the boundary to yield a nonzero solution.

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MEMORANDUM

~~Date:~~

~~To:~~
~~From:~~
~~Subject:~~

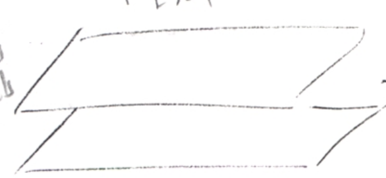
LECTURE 17

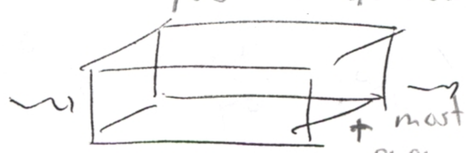
PARALLEL-PLATE WAVEGUIDES

Waveguide: a structural geometry that causes a wave to propagate in a chosen direction with some measure of confinement in the planes transverse to the propagation direction.

TEM | No TEM b/c 1 conductor | Quasi-TEM

2 plates
2 signals





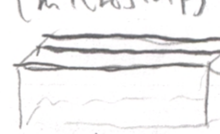
+ most energy transmitted
+ high pwr. allowed

Parallel-plate waveguide | Rectangular waveguide | Planar waveguide (microstrip)

3 plates
4 signals

N plates
2^{N-1} signals
coupling between signals

waveguides → big
→ metal loss



coplanar waveguide

Basic Equations

(·) Assume time-harmonic waves : $\sim e^{j(\omega t)-\gamma z}$

$\gamma = \alpha + j\beta$. Assume material $(\mu, \varepsilon) \to k^2 = \omega^2 \mu \varepsilon$

$$\nabla^2 \bar{E} = -k^2 \bar{E} \quad , \quad \nabla^2 \bar{H} = -k^2 \bar{H}$$

$$\nabla^2 \bar{E} = \underbrace{\nabla_t^2}_{\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}} \bar{E} + \underbrace{\frac{\partial^2}{\partial z^2} \bar{E}}_{\gamma^2 \bar{E}}$$

transverse plane to propagation

(17.2

$$\nabla_t^2 \bar{E} = -(\gamma^2+k^2)\bar{E}$$

$$\nabla_t^2 \bar{H} = -(\gamma^2+k^2)\bar{H}$$

$\Rightarrow$ $\bar{E}, \bar{H}$ fields in waveguides

$$\frac{d^2}{dz^2} E = \frac{\partial^2}{\partial z^2}\left(E(x,y)e^{-\gamma z}\right) = (-\gamma)^2 E(x,y) e^{-\gamma z}$$

Boundary conditions

* Cross section of waveguide

More convenient to solve explicitly in terms of $\boxed{\bar{E}_z, H_z}$

$\nabla \times \bar{E} = -j\omega\mu \bar{H}$, $\nabla \times \bar{H} = j\omega\varepsilon\bar{E}$ $\Rightarrow$ 6 equations

$F(E_z, H_z)$

$$E_x = -\frac{1}{\gamma^2+k^2}\left(\gamma\frac{\partial E_z}{\partial x} + j\omega\mu\frac{\partial H_z}{\partial y}\right)$$

$$E_y = \frac{1}{\gamma^2+k^2}\left(-\gamma\frac{\partial E_z}{\partial y} + j\omega\mu\frac{\partial H_z}{\partial x}\right)$$

$$H_x = \frac{1}{\gamma^2+k^2}\left(j\omega\varepsilon\frac{\partial E_z}{\partial y} - \gamma\frac{\partial H_z}{\partial x}\right)$$

$$H_y = -\frac{1}{\gamma^2+k^2}\left(j\omega\varepsilon\frac{\partial E_z}{\partial x} + \gamma\frac{\partial H_z}{\partial y}\right)$$

Propagating (Lossless) waves ($\gamma = j\beta$)

$$E_x = -\frac{j}{K_c^2}\left(\beta\frac{\partial E_z}{\partial x} + \omega\mu\frac{\partial H_z}{\partial y}\right)$$

$$E_y = \frac{j}{K_c^2}\left(-\beta\frac{\partial E_z}{\partial y} + \omega\mu\frac{\partial H_z}{\partial x}\right)$$

$$H_x = \frac{j}{K_c^2}\left(\omega\varepsilon\frac{\partial E_z}{\partial y} - \beta\frac{\partial H_z}{\partial x}\right)$$

$$H_y = -\frac{j}{K_c^2}\left(\omega\varepsilon\frac{\partial E_z}{\partial x} + \beta\frac{\partial H_z}{\partial y}\right)$$

$$K_c^2 \overset{\Delta}{=} \gamma^2 + k^2 = k^2 - \beta^2$$

$$\nabla_t^2 E_z = -K_c^2 E_z, \quad \nabla_t^2 H_z = -K_c^2 H_z$$

1. $E_z = H_z = 0 \Rightarrow$ TEM (transverse electromagnetic waves)
2. $H_z = 0 \Rightarrow$ TM (transverse magnetic or E waves)
3. $E_z = 0 \Rightarrow$ TE (transverse electric or H waves)
4. $E_z, H_z \neq 0 \Rightarrow$ Hybrid waves (coupling of TE, TM by the boundary)

STEPS IN WAVEGUIDE ANALYSIS

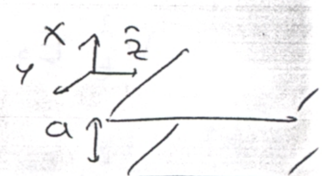
(I) Apply a general field distribution excitation

(II) Decompose in TEM, TE, TM with appropriate amplitudes and phases

(III) Study each mode in term of propagation constant

(IV) Recombine them at a later position and time to get the total fields.

Usually: we design for single-mode propagation (otherwise: dispersion, energy split)



PERFECTLY CONDUCTING PARALLEL PLATES

(a) TEM waves (plane wave)

$\gamma^2 + k^2 = 0 \Rightarrow$ propagation constant: $\gamma_{TEM} = \pm jk$

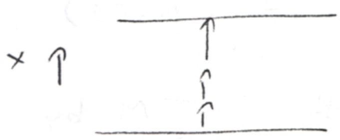
$\nabla_t^2 \bar{E} = 0 \quad \nabla_t^2 \bar{H} = 0$

(no dependence on configuration of parallel-planes) $\quad v_p = \frac{c_0}{\sqrt{\epsilon_r \mu_r}}$

↳ any TEM waves satisfy it

$k \to v_p =$ velocity of light $\Rightarrow$ propagation with light velocity

$$\nabla_t^2 \bar{E} = 0 \quad \nabla_t^2 \bar{H} = 0 \Rightarrow \text{Laplace equation (static fields)}$$



Static Electric field is constant

$E_x = E_0$ (only normal to plates)

$$H_y = \frac{\gamma}{j\omega\mu} E_x = \pm\underbrace{\sqrt{\frac{\varepsilon}{\mu}}}_{\text{constant}} E_x = \frac{1}{\eta}\hat{k}\times\bar{E}$$

(+ for $+\hat{z}$ propagation, − for $-\hat{z}$ propagation)

(!! same with plane wave expression)

2) <u>TM waves</u> $E_z \neq 0$, $H_z = 0$

$$\nabla_t^2 E_z = -k_c^2 E_z = -(\gamma^2+k^2)E_z \xrightarrow{\frac{d}{dy}=0} \frac{d^2 E_z}{\partial x^2} = -k_c^2 E_z$$

$\lambda^2 E_z = -k_c^2 E_z$, $\lambda = \pm j k_c \rightarrow C e^{jk_c x} + D e^{-jk_c x}$

$(k_c^2 = \gamma^2 + k^2) = k^2 - \beta^2$

$$\left(\frac{\partial^2}{\partial x^2} + \cancel{\frac{\partial^2}{\partial y^2}}\right) E_z = -k_c^2 E_z$$

0

$$E_z = A \sin k_c x + B \cos k_c x$$

$E_z(x=0) = E_z(x=a) = 0$ (PEC) — Boundary conditions

$E_y(x=0) = E_y(x=a) = 0$

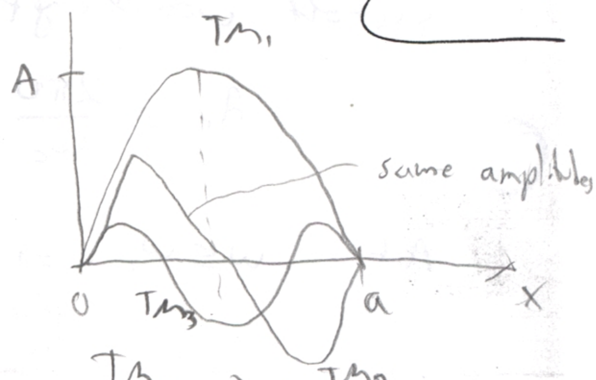
$E_z(x=0) = B\cos 0 = 0 \Rightarrow B = 0$

$E_z = A\sin(k_c x)$ and $E_z(x=a) = 0 = A\sin(k_c a)$ $\cancel{A}\neq 0$ ($E_z \neq 0$) or $\sin(k_c a) = 0$

$A = 0$ (no field) or $\sin k_c a = 0 \Leftrightarrow k_c a = m\pi$, $m = 1, 2, 3 \ldots$

$E_z = A \sin \frac{m\pi x}{a}$ $TM_m \leftarrow$ mode, $m \geq 1$

$H_z = 0$



$$H_x = \frac{1}{\gamma^2 + k^2}\left(j\omega\varepsilon \frac{\partial E_z}{\partial y} - \gamma \cancel{\frac{\partial H_z}{\partial x}}^{\,0} \right)$$

$$= 0$$

$TM_0 \rightarrow E_z = 0 \rightarrow TEM$

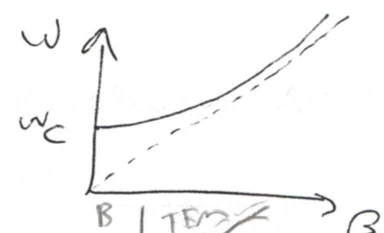
$$H_y = -\frac{j\omega\varepsilon}{k_c^2}\frac{\partial E_z}{\partial x} = -\frac{j\omega\varepsilon a}{m\pi} A \cos \frac{m\pi x}{a}$$

$$E_x = - \frac{\gamma}{k_c^2}\frac{\partial E_z}{\partial x} + 0 =$$

$$= -\frac{\gamma a}{m\pi} A \cos \frac{m\pi x}{a}$$

$$E_y = -\frac{\gamma}{k_c^2}\frac{\partial E_z}{\partial y} = 0$$

infinite number of solutions for different m (different modes)



<u>Propagation constant</u> of TM_m

$$\gamma_m = \sqrt{k_c^2 - k^2} = \boxed{\underbrace{\sqrt{\left(\frac{m\pi}{a}\right)^2}}_{C} - \omega^2\mu\varepsilon}$$

$\gamma = \alpha + j\beta$ lossless $j\omega\sqrt{\mu\varepsilon} = \sqrt{-\omega^2\mu\varepsilon}$ TEM

for $\omega > \omega_c = \frac{m\pi}{a\sqrt{\mu\varepsilon}} = \frac{m\pi v_e}{a} = \frac{m\pi}{a}\frac{c}{\sqrt{\varepsilon_r\mu_r}}$ (cutoff frequency)

$a\sqrt{\mu\varepsilon} \leftarrow$ dielectric filling v_e: velocity of light

$\omega < \omega_c \rightarrow \gamma_m = \alpha_m = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon} \rightarrow$ attenuation only

$$\gamma = j\beta = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - \frac{(m\pi/a)^2}{\omega^2\mu\varepsilon}}$$

(only propagation)

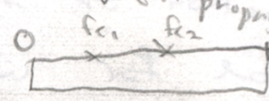
$\omega^2\mu\varepsilon > \left(\frac{m\pi}{a}\right)^2$ $f < 0$ $\gamma_m = j\beta_m = j\sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2}$

approaches TEM wave for $\omega \rightarrow \infty$

$\omega^2 > \left(\frac{m\pi}{a}\right)^2 / \mu\varepsilon$

ω so that propagation only

$$\left(\gamma = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}\right., \quad \omega \geq \omega_c$$



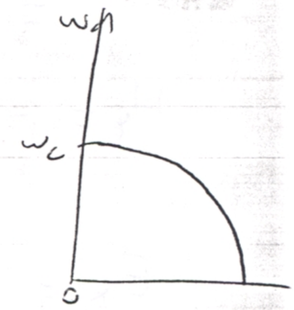
$f_{c1} \geq 28.86$ GHz

Cutoff wavelength: $v = \frac{\omega}{\beta}$

$$\lambda_c = \frac{2\pi v}{\omega_c} = \frac{2a}{m} \qquad \left(a = \frac{m\lambda_c}{2}\right)$$

At $\quad \omega \le \omega_c \Rightarrow \gamma = \alpha = \frac{m\pi}{a}\sqrt{1-\left(\frac{\omega}{\omega_c}\right)^2}$

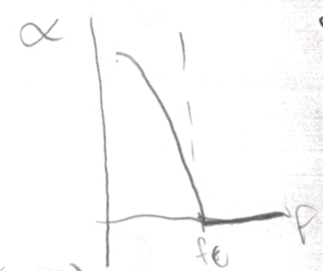
(purely attenuating wave)



(TM modes : High-pass filters)

Phase velocity:

$$v_p = \frac{\omega}{\beta} = \frac{v}{\sqrt{1-(\omega_c/\omega)^2}} > V_{light}$$



Group velocity:

$$v_g = \frac{d\omega}{d\beta} = v\sqrt{1-(\omega_c/\omega)^2} < V_{light}$$

$\omega = \omega_c \rightarrow v_g = 0$

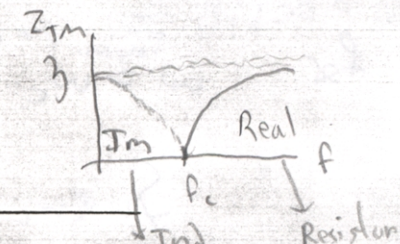
$(0, f_c) \rightarrow$ evanescent mode eliminates some pwr

Guided wavelength:

$$\lambda_g = \frac{2\pi}{\beta} = \frac{\lambda \text{ (plane wave)}}{\sqrt{1-(\omega_c/\omega)^2}} \qquad \omega = \omega_c \rightarrow \lambda_g = \infty$$

λ: wavelength of a plane in the dielectric medium

$$\lambda = \frac{2\pi v}{\omega}$$



Characteristic wave impedance:

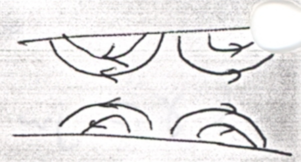
$$Z_{TM} = \frac{E_x}{H_y} = \frac{\beta}{\omega\epsilon} = \eta\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

$\eta = \sqrt{\mu/\epsilon}$ (TEM) $\qquad \omega = \omega_c \rightarrow Z_{TM} = 0$ short ckt.

function of frequency, $\neq x, y, z$

Real for $\omega \ge \omega_c \Rightarrow$ Real power transfer

Imaginary for $\omega \le \omega_c \Rightarrow$ no real power transfer



TM_1

c) TE waves

$H_z \neq 0, \quad E_z = 0$

$\chi \quad \Psi$

$$\nabla_t^2 H_z = -k_c^2 H_z = \frac{d^2 H_z}{dx^2}$$

$$k_c^2 = k^2 + \gamma^2 = k^2 - \beta^2$$

$$\left\{ E_y |_{x=0} = 0 \right.$$

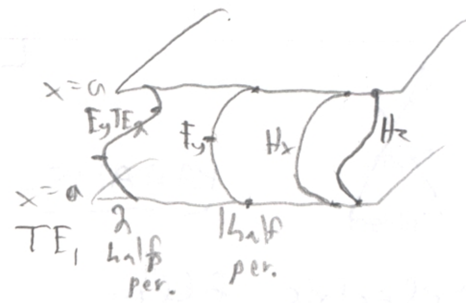
$$k_c = \frac{m\pi}{a}$$

$$H_z = B \cos k_c x$$

$$H_x = \frac{j\beta}{k_c} B \sin k_c x$$

$$E_y = -\frac{j\omega\mu}{k_c} B \sin k_c x$$

$$E_x = 0, \quad H_y = 0$$



$f_c^{TE_m} = f_c^{TM_m}$

similar to TM waves same v_p, λ, β, etc.

$$E_y|_{x=a} = 0 \Rightarrow k_c a = m\pi \Rightarrow k_c = 2\pi f_c \sqrt{\mu\varepsilon} = \frac{m\pi}{a},$$

$m = 1, 2, 3, \ldots$

$$\gamma = \begin{cases} \alpha = \left(\frac{m\pi}{a}\right)\sqrt{1 - \left(\frac{\omega}{\omega_c}\right)^2}, & \omega < \omega_c \\ j\beta = jk\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}, & \omega > \omega_c \end{cases}$$

// TM waves

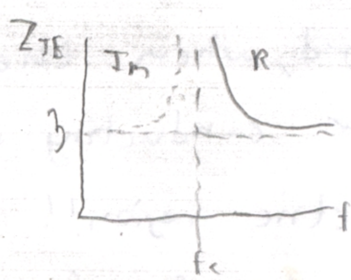
$$Z_{TE} = -\frac{E_y}{H_x} = \frac{j\omega\mu}{\gamma} = \frac{\eta}{\sqrt{1 - (\omega_c/\omega)^2}}$$

only difference

$\omega = \omega_c \rightarrow Z_{TE} = \infty$ open ckt.

Real for $\omega > \omega_c$

Imaginary for $\omega < \omega_c$



TM_1, TE_1 etc. excited in dual