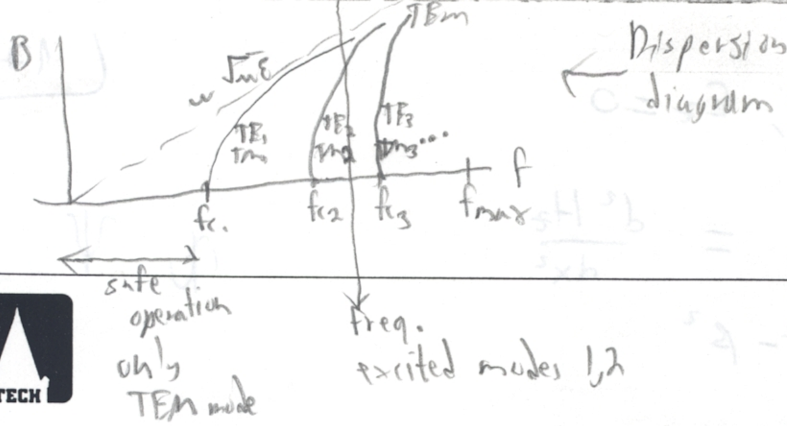


Georgia Institute of Technology
Atlanta, Georgia 30332

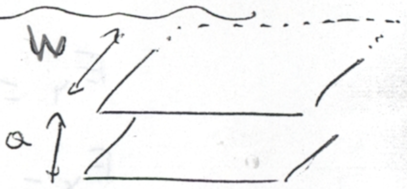
MEMORANDUM

~~Date:~~

LECTURE 18

~~To:~~
~~From:~~
~~Subject:~~

PARALLEL-PLATE WAVEGUIDE WITH LOSSES



Dielectric filling: $\varepsilon' \to \varepsilon' - j\varepsilon'' = \varepsilon' - j\frac{\sigma}{\omega}$

$$\gamma_{TEM} = j\omega\sqrt{\mu(\varepsilon' - j\varepsilon'')}$$

$$\frac{\varepsilon''}{\varepsilon'} << 1 \xLeftrightarrow{\text{Taylor}} (\alpha_d)_{TEM} \simeq \frac{\omega\sqrt{\mu\varepsilon'}}{2}\frac{\varepsilon''}{\varepsilon'} \quad (\text{1st order Approx})$$

low-loss dielectric

$$\gamma_{TM,TE} = \left[\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu(\varepsilon' - j\varepsilon'')\right]^{1/2} \approx$$

$$\approx \left[\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon'\right]^{1/2}\left\{1 + \frac{j\omega^2\mu\varepsilon''}{2}\left[\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon'\right]^{-1}\right.$$

$$\text{For } \omega > \omega_c \Rightarrow (\alpha_d)_{TM,TE} = \frac{\omega\sqrt{\mu\varepsilon'}\,(\varepsilon''/\varepsilon')}{\underbrace{2\sqrt{1-(\omega_c/\omega)^2}}_{\text{correction term to TEM}}}$$

ε, μ, σ of dielectric filling

same for both modes

$\omega = \omega_c \to \alpha = \infty$
$\omega >> \omega_c \to \alpha \cong \alpha(TEM)$

How about finite conductivity of the plats?

Ignored, when the plane boundaries are made of much better conducting material than the intervening dielectric region!!

$$\tilde{S}_{av} = \frac{E_0^2}{2\eta}$$

Power transfer for TEM: $(W_T)_{TEM} = \frac{E_0^2}{2\eta}(aw)$
(per unit length)

Attenuation constant :

$$(\alpha_c)_{TEM} = \frac{R_s}{\eta a} , \quad R_s = \sqrt{\frac{\pi f \mu}{\sigma}}$$ — metal plate

(dielectric filling of waveguides

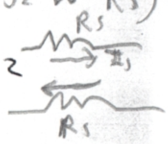
For TM_m and $\omega > \omega_c$

$$\Downarrow$$

$$(W_T)_{TM} = W \int_0^a \frac{1}{2} (E_x H_y^*) \, dx = \frac{W \omega \epsilon \beta a^2 A^2}{2 m^2 \pi^2} \frac{a}{2}$$

[A : amplitude of TM^m mode) $W_{L_{TM}} = 2\left(\frac{1}{2} I^2 R_s\right) = 2\left(\frac{1}{2} J_z^2 W^2 R_s\right)$ $\quad I = J_z W$

Average loss/unit length: $(W_L)_{TM} = W R_s \left(\frac{\omega \epsilon a A}{m \pi} \right)^2$



$\alpha = \frac{W_L}{W_T}$ $\quad \left(= \frac{2WR_s}{2} |J_{sz}|^2 = \frac{2WR_s}{2} |H_y|^2_{x=0} \right)$

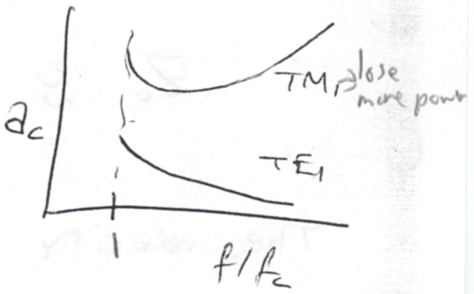
$$\Downarrow$$

Cares about 2 metals B.C.

$$(\alpha_c)_{TM} = \frac{2 R_s \omega \epsilon}{\beta a} = \frac{2 R_s}{\eta a \sqrt{1-(\omega_c/\omega)^2}}$$

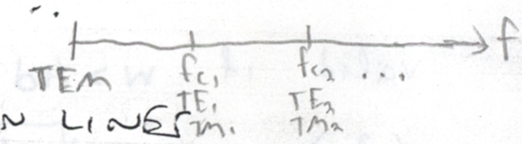
$$\searrow \quad > \quad \downarrow$$

" $$(\alpha_c)_{TE} = \frac{2 R_s (\omega_c/\omega)^2}{\eta a \sqrt{1-(\omega_c/\omega)^2}}$$



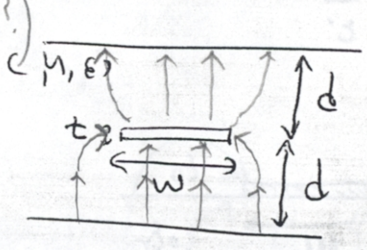
(These approximation fail as $\omega \to \omega_c$ (it should be finite))

Always $(\alpha_c)_{TM^n} > (\alpha_c)_{TE^n}$!!



PLANAR TRANSMISSION LINES

(A) stripline (What mode?)



Supports TEM wave !! $\quad \left(v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu \epsilon}} \not\propto f \right)$
$\not\propto$ thickness

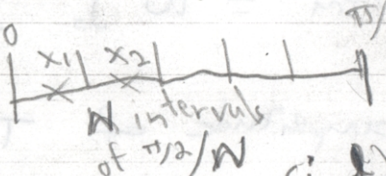
$$Z_0 = \sqrt{\frac{L}{C}} \not\propto f$$

For zero-thickness

$$Z_0 \approx \frac{\eta}{4}\frac{K_1(k)}{K_1(\sqrt{1-k^2})}, \quad \eta=\sqrt{\frac{\mu}{\varepsilon}}$$

$$k=\left[\cosh\left(\frac{\pi W}{4d}\right)\right]^{-1}, \quad K_1(k): \text{complete elliptic integral of first kind}$$

$$?.\ K_1(k)=\int_0^{\pi/2}\frac{d\varphi}{\sqrt{1-k^2\sin^2\varphi}}$$



$$\Delta\phi=\frac{\pi/2}{N}$$

$$x_i=\left(i-\frac{1}{2}\right)\Delta\phi$$

Approximation for $\frac{W}{2d}>0.56$

$$K(k)=\sum_{i=1}^{N}\frac{1}{\sqrt{1-k^2\sin^2\phi_i}}\Delta\phi$$

$$f(\phi)=\frac{1}{\sqrt{1-2\sin^3\phi}}$$

$$K(k)=\sum f(\phi=x_i)\Delta\phi$$

$i=1\to N$

$$Z_0 \approx \frac{\eta\pi}{8\ln[2\exp(\pi W/4d)]}$$

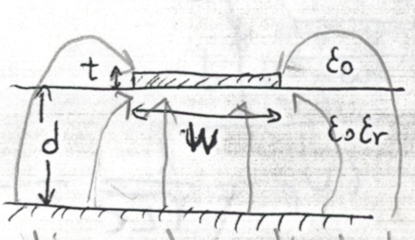
The velocity of propagation $\neq$ thickness of conductor $\left(\frac{1}{\sqrt{\mu\varepsilon}}\right.$

$$\alpha_c=\frac{R_s}{2\eta d}\left[\frac{\pi W/2d+\ln(8d/\pi t)}{\ln 2+\pi W/4d}\right] \text{ Np/m}$$

valid if $W>4d$ and $t<d/5$

$$(\alpha_d)_{TEM}\simeq\frac{\omega\sqrt{\mu\varepsilon'}}{2}\frac{\varepsilon''}{\varepsilon'}$$

(B) Microstrip



Cannot support a pure TEM wave!?

$\Updownarrow$

1/2 lines above, 1/2 below $\to \gamma^2+k^2=0$

different v_p in mediums

for $\varepsilon_r\neq 1 \Rightarrow k_1\neq k_2 \Rightarrow$ cannot have the same γ

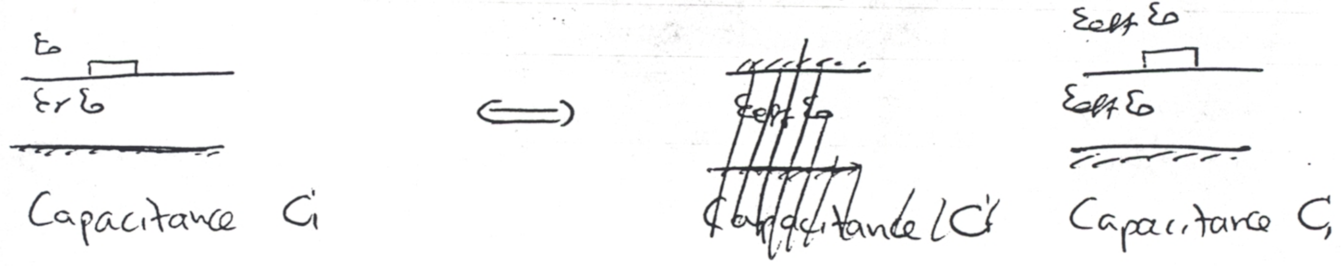
Quasi-TEM mode

18.4

Char. Impedance for zero metal thickness and free space dielectric:

$$Z_{00} = 377\left[\frac{W}{d} + 1.98\left(\frac{W}{d}\right)^{0.172}\right]^{-1}$$

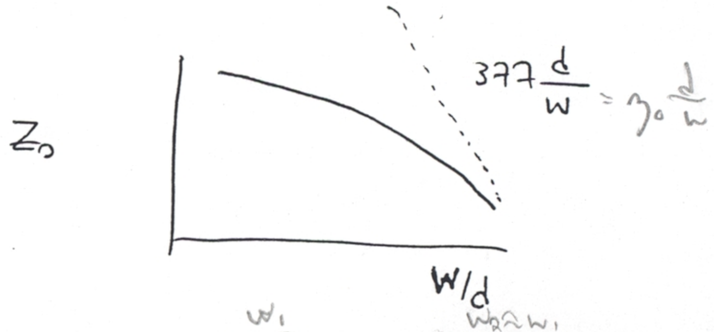
(Accuracy < 0.3% for W/d > 0.06)



$$\varepsilon_{eff} = 1 + \frac{(\varepsilon_r - 1)}{2}\left[1 + \frac{1}{\sqrt{1+10\,d/W}}\right] \quad (*)$$

$$(Z_0 = Z_{00} / \sqrt{\varepsilon_{eff}})$$

$$u_p = \frac{c_0}{\sqrt{\varepsilon_{eff}}}$$

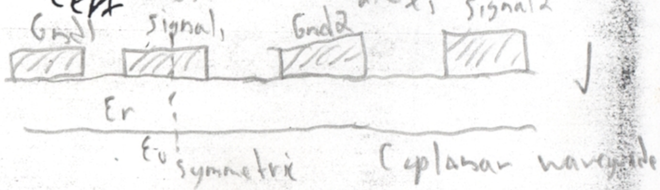


Above:

$$f_{max} = \frac{21 \times 10^6}{(W + 2d)\sqrt{\varepsilon_r + 1}}$$

there is a frequency dependence of ε_{eff}

$$\sqrt{\varepsilon_{eff}(f)} = \frac{\sqrt{\varepsilon_r} - \sqrt{\varepsilon_{eff}(0)}}{1 + 4F^{-1.5}} + \sqrt{\varepsilon_{eff}(0)}$$



$\varepsilon_{eff}(0) = (*)$

$$F = \frac{4fd\sqrt{\varepsilon_r - 1}}{c_0}\left\{0.5 + \left[1 + 2\ln\left(1 + \frac{W}{d}\right)\right]^2\right\}$$

ductors $R_s \rightarrow \alpha_c = (R_s\sqrt{\varepsilon_{eff}(0)}/d) \cdot \textcircled{A} \rightarrow$ width-thickness parameter Wt

dielectric $\tan\delta_\varepsilon$ $(= \varepsilon''/\varepsilon')$ $\sim \alpha_\varepsilon = \dfrac{\pi f \tan\delta_\varepsilon}{c_0}\sqrt{\dfrac{\varepsilon_r(1+F_1)}{2}}\left[1 + \dfrac{1-F_1}{\varepsilon_r(1+F_1)}\right]^{-1}$ NP/m

$F_1 = [1 + (10\,d/W)]^{-1/2}$ Also variation

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~~To:~~
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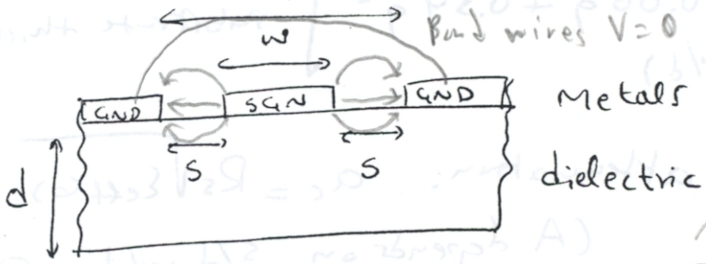
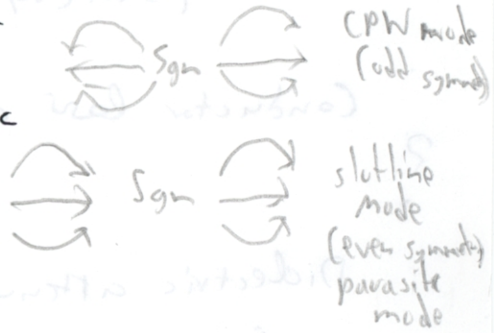
LECTURE 19

Coplanar Waveguide

better for higher f





Quasi-TEM mode (2 dielectrics)

$$\varepsilon_{eff} = \frac{\varepsilon_r+1}{2}, \quad v_p = \frac{1}{\sqrt{\mu_1 \varepsilon_0 \varepsilon_{eff}}}$$

For zero-thickness conductors, infinitely wide GND conductors and infinite substrate thickness:

$$Z_0 = \eta_0 \frac{K_1(k')}{4\sqrt{\varepsilon_{eff}}\, K_1(k)}, \quad k = \frac{w}{a}, \quad k' = \sqrt{1-k^2}$$

Approximations (<0.25% error)

$$Z_0 = \begin{cases} \dfrac{\eta_0}{\pi\sqrt{\varepsilon_{eff}}} \ln\left(2\sqrt{\dfrac{a}{w}}\right), & 0 < \dfrac{w}{a} < 0.173 \\[2ex] \dfrac{\pi\eta_0}{4\sqrt{\varepsilon_{eff}}} \left[\ln\left(2\dfrac{1+\sqrt{w/a}}{1-\sqrt{w/a}}\right)\right]^{-1}, & 0.173 < \dfrac{w}{a} < 1 \end{cases}$$

if d is finite and >> a, these expressions are accurate within 5%

②

Better dispersion characteristics than μstrip

$$\sqrt{\varepsilon_{eff}(f)} = \frac{\beta}{k_0} = \frac{\sqrt{\varepsilon_r} - \sqrt{\varepsilon_{eff}(0)}}{1 + b F_2^{-1.8}} + \sqrt{\varepsilon_{eff}(0)}$$

$F_2 = 2fd\sqrt{\varepsilon_r - 1} / c_0$, $\varepsilon_{eff}(0) = \frac{\varepsilon_r + 1}{2}$

$b = \exp[u \ln(w/s) + r]$

$u = 0.54 - 0.64q + 0.015q^2$
$r = 0.43 - 0.86q + 0.54q^2$
$q = \ln(w/d)$

} depend on substrate thickness

?

Conductor loss attenuation: $a_c = R_s \sqrt{\varepsilon_{eff}(0)} A/d$

(A depends on s/d, w/d – Fig 8.6d)
curve

Dielectric attenuation: $a_d = \frac{\pi f \sqrt{\varepsilon_{eff}(0)}}{c_0} \left[\frac{1 - 1/\varepsilon_{eff}(0)}{1 - 1/\varepsilon_r} \right] \tan\delta_\varepsilon$, Np/m
(d/a >1)

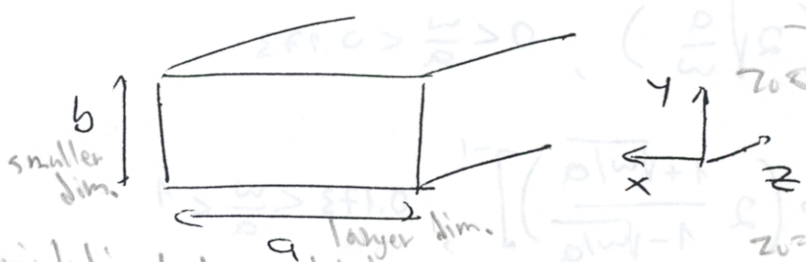
$\frac{\varepsilon''}{\varepsilon'}$

Slot line

not symmetrical
vulnerable to parasitics

Coplanar strip

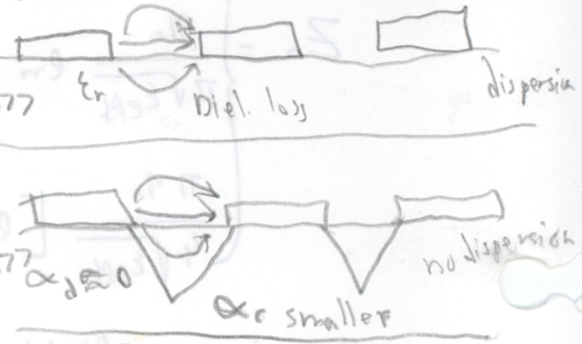
Rectangular Waveguide



better isolation, higher metal loss

No TEM!! b/c 1 PEC
No DC waves
low f. waves

Micromachined Tr-lines (Membrane)

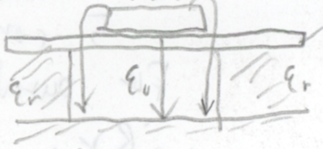


Bulk/Volume
Micromachined line $Z_0 = \sqrt{\frac{\mu}{\varepsilon}} = 377$ air

$Z_0(\varepsilon_r) < 377$

$\alpha_c \propto I^2 = \frac{V^2}{Z_0^2}$ $v_p \propto \frac{c_0}{\sqrt{\varepsilon_{eff}(f)}}$

μ-strip



surface/membrane μmachined line

TM $(H_z = 0,\ E_z \neq 0)$

$$\nabla_t^2 E_z = \frac{\partial^2 E_z}{\partial x} + \frac{\partial^2 E_z}{\partial y} = -k_c^2 E_z$$

Separation of variables

$$E_z = (A' \sin k_x x + B' \cos k_x x)(C' \sin k_y y + D' \cos k_y y)$$

$$k_x^2 + k_y^2 = k_c^2$$

$E_z|_{x=0} = 0 \Rightarrow B' = 0$

$E_z|_{y=0} = 0 \Rightarrow D' = 0$

$\xrightarrow{A'C' = A}$ $E_z = A \sin k_x x \cdot \sin k_y y$

$A = 0 \rightarrow$ TEM wave

$E_z|_{x=a} = 0 = A \sin(k_x a) \sin(k_y y) \Rightarrow k_x a = m\pi$, $m = 1, 2, 3 \ldots$

$E_z|_{y=b} = 0 \Rightarrow k_y b = n\pi$, $n = 1, 2, 3 \ldots$

$$E_z^{TM_{mn}}(x,y) = A \sin\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right)$$

→ Cutoff:

$$\omega_{c_{m,n}} = \frac{k_{c_{m,n}}}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} \underbrace{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}}_{k_c^{TM_{mn}}}$$

$$\left\{ k_c^2 = k^2 - \beta^2 \right.$$

$$\gamma = \alpha = k_{c_{m,n}} \sqrt{1 - \left(\frac{\omega}{\omega_{c_{m,n}}}\right)^2}, \quad \omega < \omega_{c_{m,n}}$$

$$\gamma = j\beta = jk \sqrt{1 - \left(\frac{\omega_{c_{m,n}}}{\omega}\right)^2}, \quad \omega > \omega_{c_{m,n}}$$

Velocities similar to Parallel-plate.

5 components ≠ 0
For parallel → 3 comp. ≠ 0

$$E_x = -\frac{j\beta k_x}{k_{c_{m,n}}^2} A \cos \underset{\left(\frac{m\pi}{a}\right)}{k_x} x \sin \underset{\left(\frac{n\pi}{b}\right)}{k_y} y$$

$$E_y = -\frac{j\beta k_y}{k_{c_{m,n}}^2} A \sin k_x x \cos k_y y$$

$$H_x = \frac{j\omega\varepsilon k_y}{k_{c_{m,n}}^2} A \sin k_x x \cos k_y y$$

$$H_y = -\frac{j\omega\varepsilon k_x}{k_{c_{m,n}}^2} A \cos k_x x \sin k_y y$$

$$\cdot e^{-j\beta z}$$

same space dependence

Conductor Losses:

$$(\alpha_c)_{TM_{m,n}} = \frac{2R_s}{b\eta\underbrace{\sqrt{1-(f_c/f)^2}}_{\text{similar to parallel}}} \frac{[m^2(b/a)^3+n^2]}{[m^2(b/a)^2+n^2]}$$ ~ ratio of surface to volume

⚠ $m,n \neq 0$ always !!

// TE waves ($E_z=0$, $H_z\neq0$)

$$H_z = B' \cos \underset{\frac{m\pi}{a}}{k_x} x \cos \underset{\frac{n\pi}{b}}{k_y} y$$

$k_x a = m\pi$

$k_y b = n\pi$

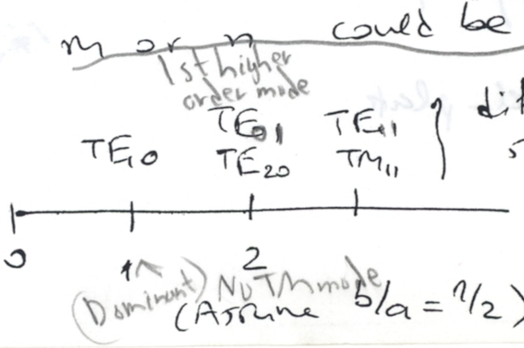
$w_c^{TE_{mn}} = w_c^{TM_{mn}}$

	m	n	w_c
TE_{10}	1	0	$\frac{\pi}{a}\frac{1}{\sqrt{\mu\varepsilon}}$
TE_{20}	2	0	$\frac{2\pi}{a}\frac{1}{\sqrt{\mu\varepsilon}}$
TE_{11} TM_{11}	1	1	$\frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2}$

→ a>b

or H_z = constant plane wave (TEM)

Now!. m or n could be zero (not both of them)



} different field distribution
same cutoff frequency
↓
degenerate modes

⑤

$$E_x = \frac{j\omega\mu k_y}{K_{c_{m,n}}^2} B \cos k_x x \sin k_y y$$

$$E_y = -\frac{j\omega\mu k_x}{K_{c_{m,n}}^2} B \sin k_x x \cos k_y y$$

$$H_x = \frac{j\beta k_x}{K_{c_{m,n}}^2} B \sin k_x x \cos k_y y$$

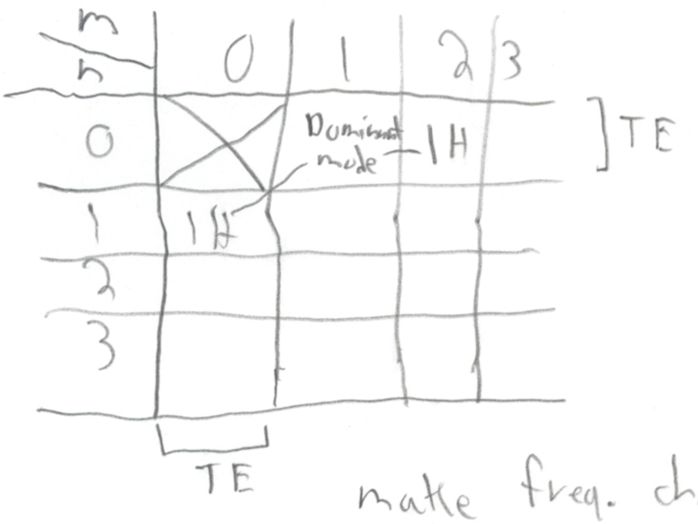
$$H_y = \frac{j\beta k_y}{K_{c_{m,n}}^2} B \cos k_x x \sin k_y y$$

similar
to TM
but diff. constant coeff.

$k_x, k_y, \omega_{c_{m,n}}$ and α, β have the same form for $TE_{m,n}/TM_{m,n}$!!

For TE modes ⇒ small b/a ratios give large attenuation because of the high ratio of surface to cross-sectional area.

Calc. fc's.



m / n	0	1	2	3
0		1H		
1	1H			
2				
3				

make freq. chart

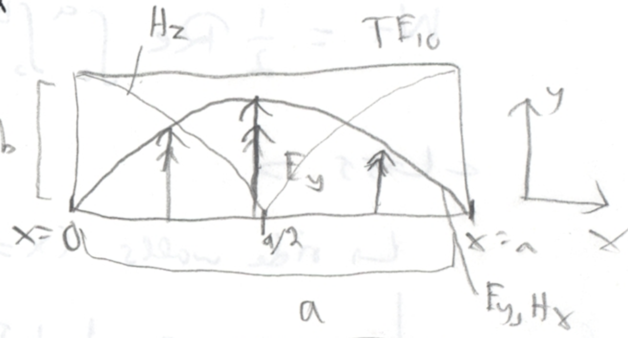
For high freq. operation of chip, excite several modes and disperse energy.

LECTURE 20

TE_{10} IN A RECTANGULAR GUIDE

- Cutoff frequency is independent of one of the dimensions of the cross section.
- The polarization of $\bar{E}$ is fixed (from top to bottom)
- Metal loss is comparable to other lines

$m=1,\ n=0 \Rightarrow k_y = 0,\quad k_x = k_c = \pi/a$



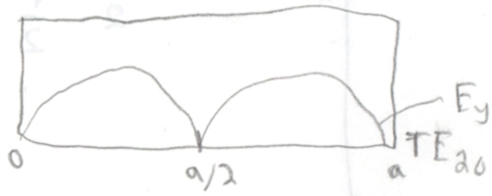
$$H_z = B \cos k_x x$$

$$E_y = -\frac{j\omega\mu B}{k_x} \sin k_x x$$

$$H_x = \frac{j\beta B}{k_x} \sin k_x x$$

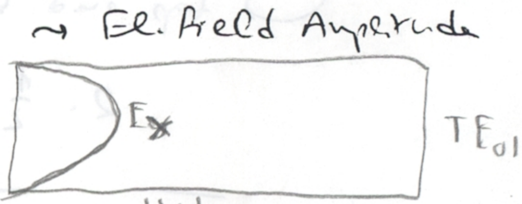
propagating constant

$$E_x = E_z = H_y = 0$$



To define the mode impedance:

$$E_y = -Z_{TE} H_x = E_0 \sin\left(\frac{\pi x}{a}\right)$$



$$H_z = \frac{jE_0}{\eta}\left(\frac{\lambda}{2a}\right)\cos\left(\frac{\pi x}{a}\right)$$

$$E_0 = -\frac{j\omega\mu B}{k_x} = -\frac{j2\eta a B}{\lambda}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}, \quad \lambda = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}}$$

$$Z_{TE} = \eta\left[1-\left(\frac{\omega_c}{\omega}\right)^2\right]^{-1/2} = \eta\left[1-\left(\frac{\lambda}{2a}\right)^2\right]^{-1/2}$$

$Z_{TM} = \eta\sqrt{\quad}$

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}}\,\frac{1}{\sqrt{1-(\lambda/2a)^2}}, \quad v_g = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}$$

$$\lambda_g = \frac{v_p}{f} = \frac{2\pi}{\beta} = \lambda \Big/ \sqrt{1-(\lambda/2a)^2}$$

②

Cutoff frequency: $f_c = \frac{1}{2a\sqrt{\mu\varepsilon}}$, $\lambda_c = 2a$

$\uparrow$
Cutoff wavelength

Dielectric Attenuation: $(\varepsilon' - j\varepsilon'' = \varepsilon)$

$$\alpha_d = \frac{k\varepsilon''/\varepsilon'}{2\sqrt{1-(\lambda/2a)^2}} \qquad k = \omega\sqrt{\mu\varepsilon}$$

Metal Attenuation:

▲ < Transfered Power >

$$W_T = \frac{1}{2}\mathrm{Re}\int_0^a\int_0^b (-E_y H_x^*)\, dx\, dy = \boxed{\frac{E_0^2\, ba}{4 Z_{TE}}}$$

< Loss >

↳ side walls $(x=0, x=a)$ due to $|J_{sy}| = |H_z|$

$$2 \cdot \frac{1}{2} b R_s |H_z|^2_{x=0} = \frac{bR_s E_0^2 \lambda^2}{4\eta^2 a^2}$$

↳ top and bottom $(y=0, y=b)$ due to $|J_{sx}|, |J_{sz}|$

$$2 \cdot \frac{1}{2} R_s \int_0^a (|H_x|^2 + |H_z|^2)\, dx = \frac{a}{2} R_s \left(\frac{E_0^2}{Z_{TE}} + \frac{E_0^2\lambda^2}{4\eta^2 a^2}\right)$$

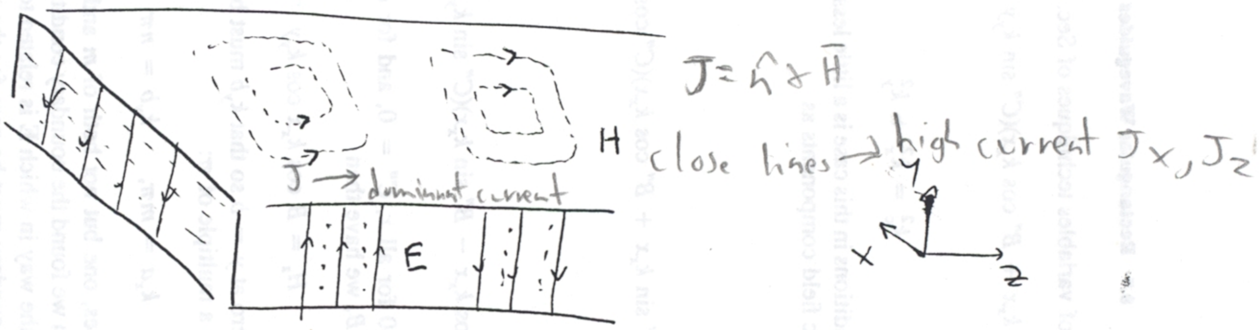
$Z_{TE_{10}, TM_{mn}}$

TE or TM ☆ Total: $\alpha_c = \frac{R_s \cancel{E_0^2}}{\underset{ba}{2}\eta^2}\left(a + \frac{b\lambda^2}{2a^2}\right)$

$\Updownarrow$

$$☆\ \alpha_c = \frac{R_s}{b\eta\sqrt{1-(\lambda/2a)^2}}\left[1 + \frac{2b}{a}\left(\frac{\lambda}{2a}\right)^2\right]$$

Field patterns



No field components vary in the vertical (y) direction

Induced charges induced by el. field lines ending on conductors are zero on side walls and $\rho_s = \varepsilon E_y$ on bottom and $-\varepsilon E_y$ on top plate.

Why b plays no role in determining TE_{10} cutoff freq?!

Always $\bar{E}$ is normal to top and bottom plates

↳ they do not contribute any boundary condition.

BUT

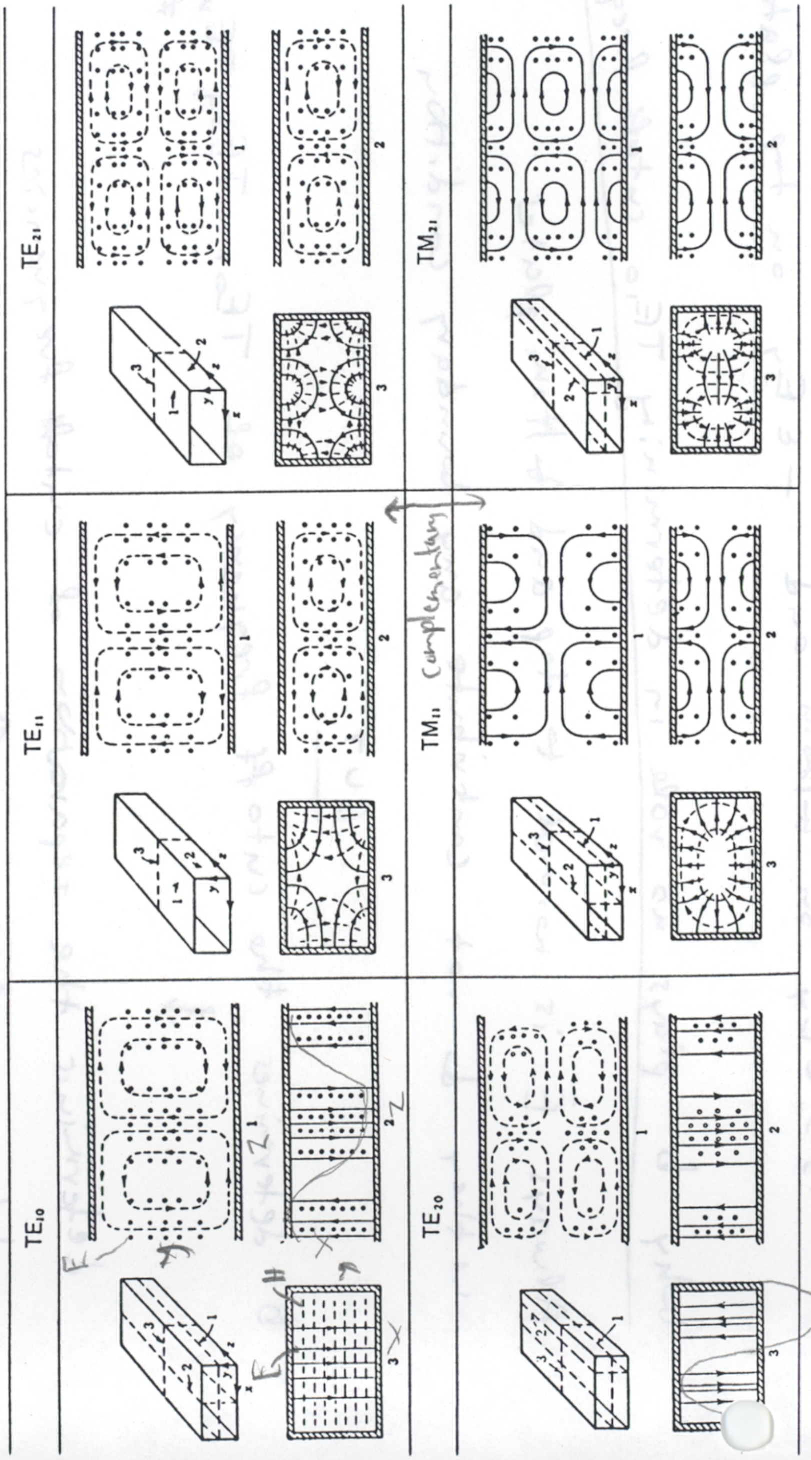
b determines the cutoff frequency of TE_{01}, TE_{02}, TE_{mn} $\neq 0$

↓

determines the separation of cutoff frequencies

b↓ → { separation ↑
~~loss~~ attenuation ↑
power handling ↓

Table 8.7
Summary of Wave Types for Rectangular Guides[a]



[a] Electric field lines are shown solid and magnetic field lines are dashed.

Solution by the separation of variables techniques of Sec. 7.19 gives

$$H_z = (A'' \sin k_x x + B'' \cos k_x x)(C'' \sin k_y y + D'' \cos k_y y) \qquad ($$

where

$$k_c^2 = k_x^2 + k_y^2 \qquad ($$

Imposition of boundary conditions in this case is a little less direct, but from Eqs. 8.2(and 8.2(14) we find electric field components as

$$E_x = -\frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y}$$

$$= -\frac{j\omega\mu k_y}{k_c^2}(A'' \sin k_x x + B'' \cos k_x x)(C'' \cos k_y y - D'' \sin k_y y) \qquad (1$$

$$E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x} \qquad (1$$

$$= \frac{j\omega\mu k_x}{k_c^2}(A'' \cos k_x x - B'' \sin k_x x)(C'' \sin k_y y + D'' \cos k_y y)$$

For E_x to be zero at $y = 0$ for all x, $C'' = 0$, and for $E_y = 0$ at $x = 0$ for all $A'' = 0$. Defining $B''D'' = B$, we have then

$$H_z = B \cos k_x x \cos k_y y \qquad (20$$

We also require E_x to be zero at $y = b$ so that $k_y b$ must be a multiple of π. E_y is zer at $x = a$ so that $k_x a$ is also a multiple of π:

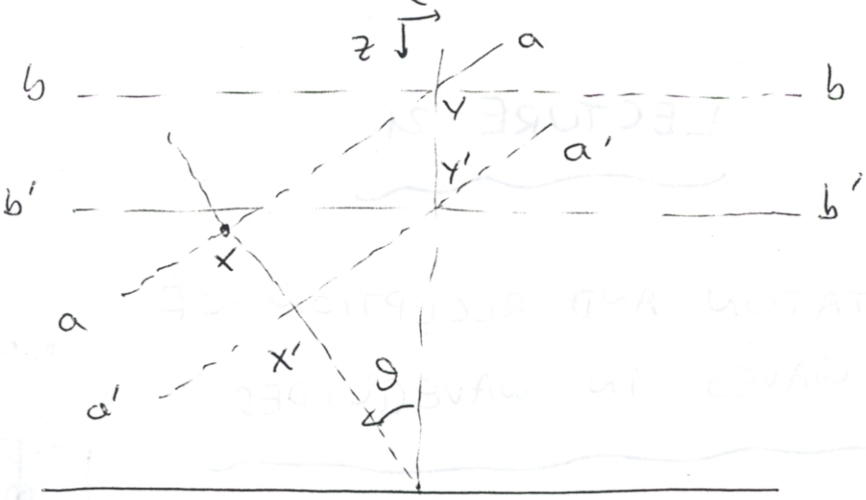
$$k_x a = m\pi, \qquad k_y b = n\pi \qquad (21$$

In contrast to the TM waves, one but not both of m and n may be zero without th wave's vanishing. Although we found the boundary conditions by first calculating elec tric field, we can see from the way in which **E** is related to H_z that the derivative of H normal to the conducting boundary must be zero for the tangential electric field to be zero there, so boundary conditions can be imposed directly on the form (16) withou requiring the explicit forms for E_x and E_y.

The forms of transverse electric field with the derived simplifications to (18) and (19) are

$$E_x = \frac{j\omega\mu k_y}{k_{c_{m,n}}^2} B \cos k_x x \sin k_y y \qquad (22)$$

$$E_y = -\frac{j\omega\mu k_x}{} B \sin k_x x \cos k_y y$$



If a plane of constant phase aa moves to a'a' in a given interval of time, the distance moved normal to the wavefront is XX', but the distance moved by this constant phase difference along the z-direction is $YY' = XX'/\cos\vartheta$

TOTAL REFLECTION – CRITICAL ANGLE

$|Z_L|_{TM} = 0 \leftrightarrow \sin\vartheta_c = \frac{v_1}{v_2} \overset{\mu_1=\mu_2}{=\!=} \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$ (only when $\varepsilon_1 > \varepsilon_2$)

The angle of refraction is $\frac{\pi}{2}$ for $\vartheta = \vartheta_c$ and is imaginary for greater angles of incidence

$\Downarrow$

no transfer of energy to the second medium

$\Downarrow$

fields die off exponentially with distance from boundary

($e^{-j\beta_z z}$, $\beta_z = k_2\cos\vartheta_2 = k_2\sqrt{1-\left(\frac{v_2}{v_1}\right)^2\sin^2\vartheta_1}$ for medium 2

β_z becomes imaginary ($e^{-\alpha z}$) when $\frac{v_2}{v_1}\sin\vartheta_1 > 1$

$\Downarrow$

$\alpha = k_2\sqrt{\left(\frac{v_2}{v_1}\right)^2\sin^2\vartheta_1 - 1}$ Np/m , $k_2 = \frac{2\pi\sqrt{\varepsilon_{r2}}}{\lambda_0}$