The cutoff frequency is lowered because of the capacitive effect at the center. It can be further decreased as well applied to nonuniform trapezoidal for matching purposes.

At zero transverse cutoff, there is no variation to $z$-direction ($\delta z = 0$) giving resonant condition for waves propagating only transversely in the given cross section.

Approximation by $\Delta / 20$ (dependence only on $x$-direction)

Gap $c \approx$ capacitance

Side sections $\approx$ One-turn solenoidal inductance

$$c_{0} = \frac{1}{2 \pi} \left( \frac{8 l_A}{\pi} \right)^{1/2} = \frac{1}{2 \pi} \left( \frac{2 d_e}{d} \right)^{1/2} \left( \frac{\pi l h}{2} \right)^{1/2}$$

$$= \frac{1}{2 \pi} \left( \frac{8}{\pi l h d} \right)^{1/2}$$

Good approximation for smaller gaps.

(Transverse Resonance Frequency Technique)

(-) Decreased power-handling capability

(+) Smaller dimensions than rectangular waveguides
A surface that possesses a reactive surface impedance keeps the fields localized on it. Surface-guiding thus allows the energy to be maintained near the surface and it is not radiated or coupled seriously to nearby objects.

\[ E_z(x, t) = e^{j(k_x x + k_y y + \omega t)} \]

**TM wave**

\[ E_x = 0, \quad H_y = \frac{\omega}{k_x} D \sin k_x (x + d) \]

\[ E_z = \frac{j \beta D}{k_x} \cos k_x (x + d) \]

\[ k_x = k_x^0 - \frac{\beta}{c^2} \]

**Field impedance at \( x = 0 \):**

\[ jX = \left. \frac{E_z}{H_y} \right|_{x = 0} = \frac{j k_x}{c} \tan k_x d, \quad \text{imaginary} \]

\[ \text{Inductive} \]

On surface guided waves, the phase velocity is less than the velocity of light in the extraneous dielectric and greater than that of the guided wave.

Similarly:

\[ \frac{d}{dx} \left( \frac{E_z}{H_y} \right) = j \omega \tan k_x d, \quad k_x = \omega \sqrt{\varepsilon} \]

\[ \varepsilon = \sqrt{\varepsilon_0} \]

\[ j \beta^2 \]

\[ k_x = \omega \sqrt{\varepsilon} \]
Parallel-plane transmission line with one-side periodic loading.
Narrow troughs disturb bottom y-directed currents, introduce series inductors and capacitance branches in series with line conductors.

constant \( E_z, H_y \) across the gap width \( w \)

\[
\frac{\partial}{\partial y} = 0
\]

consider wave: \( E_x, E_z, H_y \) \((TM)\)

\( \text{Fresnel, } E_z = \text{a periodic structure w/period } d \)

\( \text{propagation constant } \beta \text{ TM waves } (n = \omega \mu \varepsilon_0) \)

\[
\begin{align*}
E_z(x, z) & = \sum_n A_n \sin k_n (a - x) e^{-j\beta_n z} \\
E_x(x, z) & = \sum_n \frac{j \beta_n}{k_n} A_n \cos k_n (a - x) e^{-j\beta_n z} \\
H_y(x, z) & = \sum_n j \omega \mu \varepsilon_0 \frac{k_n}{k_n} A_n \sin k_n (a - x) e^{-j\beta_n z}
\end{align*}
\]

\( k_n^2 = \omega^2 \mu \varepsilon_0 - \beta_n^2 = k_n^2 - \beta_n^2 \)

\( E_z(0, z) \) is a periodic function of \( z \) analyze for some w/ diffraction

\( 1) \ E_z(0, z) = e^{-j\beta_0 z} \sum_n C_n e^{-j\frac{2\pi n}{d}z} \)

\( C_n = \frac{1}{d} \int_0^{d/2} E_z(0, z) e^{j(\beta_n - \beta_0)z} e^{-j\frac{2\pi n}{d}z} dz = \frac{E_0}{\pi m} \sin \left( \frac{n\pi w}{d} \right) \)

\( E_x(x, z) \) and \( H_y(x, z) \) additive. Capacitors and inductors (filter).
\[ A_n = \frac{E_0}{\pi n} \frac{\sin(\pi n \omega / d)}{\sin k_n a} \]

\[ \beta_n = \beta_0 + \frac{2 \pi n}{d} \]

Here, TM modes are coupled by the periodic boundary condition and have to coexist ("spatial harmonics").

Assume that the trend is approximated by a short:

\[ \frac{d}{c} = \cot k \ell \]

\[ \frac{1}{2} \frac{d}{c} \cot k \ell = \sum \frac{j \omega}{\pi n K_n} \sin \frac{\pi n \omega}{d} \cot K_n a \]

\[ \sqrt{k_n^2} = k - \beta_n^2 \]

\[ \beta_n = \beta_0 + \frac{2 \pi n}{d} \]

\[ \beta_0 = \ldots \]

\[ \text{Lumped equiv. circuit (} f_{2d} < f_1) \]

\[ C = \varepsilon \frac{d - w}{a} \]

\[ (L_1 + L_2) / 2 \quad \varepsilon L_1 = \mu a (d - w) \quad \text{between} \quad f_{2d} \quad \text{and} \quad L_2 = \frac{\pi w}{\omega} \tan k \ell \quad \text{(S.C.)} \]
\[ \sum_{n=0}^{\infty} + \sum_{n=-\infty}^{0} \]

Group velocities of all modal harmonics is equal:

\[ \frac{1}{\nu_n} = \frac{d\beta_n}{d\omega} = \frac{d\beta_0}{d\omega} \]

Floquet's theorem (Fundamental for the study of periodic structures)

Field distribution at plane \( z = nd \) is the same as at \( z = 0 \), except for factor \( e^{-j\beta_0 nd} \)

\[ E_z(x, y, z) = e^{-j\beta_0 nd} E_z(x, y, 0) \]

(7) The wave is cut off where \( \nu = 0 \)

Above this frequency, increase (finite) attenuation

**But**: other pass bands exist at higher frequencies.
Resonant circuits

(a) Lumped-element type

- Capacitor for storage of E-energy
- Inductor for storage of H-energy

At resonance, there is exchange of energy

(b) Distributed type

Standing wave set up by interference between forward and backward traveling waves (Shorts)

Cavity - stores energy

RECTANGULAR CAVITY - handles more power

Resonant frequency:

\[ f_{\text{res}} = \frac{c}{2} \sqrt{\frac{1}{a^2 + \frac{d^2}{4}}} \]

Equation of a TE_{10} mode:

- Electric field: \( E_y = 0 \) at \( x = 0, z = d \)
- Magnetic field: \( H_x = \frac{1}{\mu_0 c} (E_+ e^{-j\beta_x} + E_- e^{j\beta_x}) \sin \frac{\pi x}{a} \)

- Electric field: \( E_z = \frac{1}{\epsilon_0 c} (E_+ e^{-j\beta_z} - E_- e^{j\beta_z}) \sin \frac{\pi z}{a} \)
\[ H_2 = \frac{j \alpha}{q} \left( \frac{1}{2a} \right) (E_y - e^{-j\beta_z} + E_z - e^{j\beta_z} \sin \left( \frac{\pi x}{a} \right) + (E_y - e^{-j\beta_z}) \cos \left( \frac{\pi x}{a} \right) \right) \]

\[ E_y = 0, \quad E_z = -E_x \]

\[ F_0 = -2jE_x = E_0 \sin (2\pi a) \sin \left( \frac{\pi x}{a} \right) \]

\[ E_0 = 0, \quad z = \frac{1}{\sin \left( \frac{\pi x}{a} \right)} \]

\[ E_x = F_0 \sin \left( \frac{\pi x}{a} \right) \sin \left( \frac{\pi z}{d} \right) \]

\[ H_x = -j \frac{E_0}{\tau} \frac{1}{2d} \sin \left( \frac{\pi x}{a} \right) \cos \left( \frac{\pi z}{d} \right) \]

\[ H_z = j \frac{E_0}{\tau} \frac{1}{2d} \cos \left( \frac{\pi x}{a} \right) \sin \left( \frac{\pi z}{d} \right) \]

Field patterns:

\[ E_x = 0 \Rightarrow H_x = E_x \]

The total energy passes between electric and magnetic fields:

\[ U = \langle W_E \rangle_{\text{max}} = \frac{1}{2} \int_0^b \int_0^a \int_0^d E_x^2 \, dx \, dy \, dz = \frac{\varepsilon \omega d}{2} E_0^2 \text{ Vol.} \]

Power loss equation:

\[ \text{Fronts: } J_{sy} = -H_x, \quad \text{Back: } J_{sy} = H_x, \quad z = 0 \]

\[ \text{Left: } J_{sx} = -H_z, \quad \text{Right: } J_{sx} = H_z, \quad x = a \]

\[ \text{Top: } J_{sx} = -H_z, \quad \text{Bottom: } J_{sx} = H_z, \quad z = H_x \]

If the conducting wires have resistivity \( R_3 \), then:

\[ p = \frac{1}{2} R_3 R_z \]

\[ p = p \left( \sum_j \left[ \int_0^b \int_0^a \int_0^d H_x^2 \, dx \, dy \, dz + 2 \int_0^a \int_0^b \int_0^d H_z^2 \, dx \, dy \, dz \right] \right) \]
\[ W_L = \frac{R_s}{\varepsilon_0 \eta^2} E_0 \left[ \frac{ab}{d^2} + \frac{bd}{a^2} + \frac{1}{2} \left( \frac{a}{d} + \frac{1}{d} \right) \right] \]

\[ Q = \frac{W_Q U}{W_L} \]

\[ U \sim \frac{V}{A t} \]

\[ Q = \frac{\pi \eta}{4 R s} \left[ \frac{2b (a^2 + d^2)}{a d (a^2 + d^2) + 2bd(a^2 + b^2)} \right] \]

For cube: \(a = b = d\) \(\Rightarrow Q \approx 0.742 \frac{\eta}{R_s}\)

Usually, very low \(Q\) in comparison to lumped resonators (\(\sim 10^3\)s)

Dielectric losses and radiation from small holes lower \(Q\).

Bandwidth of a resonator

\[ \frac{\Delta f}{f_0} \approx \frac{1}{Q} \]

\(\Delta f\): distance between points on the response curve for which amplitude response is down to \(\frac{1}{2}\) of its maximum value ("half-power" points)
For a rectangular waveguide, the TEM mode is

\[ H_2 = (Ae^{-j\beta z} + Be^{j\beta z}) \cos \frac{m \pi x}{a} \cos \frac{n \pi y}{b} \]

In the resonators: \( H_2 \equiv 0 \mid z = a, b \)

\[ B = -A, \quad \beta d = \frac{m \pi}{a}, \quad \beta d = \frac{n \pi}{b} \]

\[ H_2 = A (e^{-j\beta z} - e^{j\beta z}) \cos \frac{m \pi x}{a} \cos \frac{n \pi y}{b} = \]

\[ = -A 2j \sin (\beta z) \cos \frac{m \pi x}{a} \cos \frac{n \pi y}{b} = \]

\[ = -2jA \sin (\frac{m \pi x}{a}) \cos (\frac{n \pi y}{b}) \sin (\frac{\beta d}{d}) \]

\[ H_x = -\frac{C}{k_c^2} \left( \frac{\pi}{d} \right) \left( \frac{m \pi}{a} \right) \sin (\frac{m \pi x}{a}) \cos (\frac{n \pi y}{b}) \cos (\frac{\beta d}{d}) \]

\[ H_y = -\frac{C}{k_c^2} \left( \frac{\pi}{d} \right) \left( \frac{n \pi}{b} \right) \cos (\frac{m \pi x}{a}) \sin (\frac{n \pi y}{b}) \cos (\frac{\beta d}{d}) \]

\[ E_x = \frac{jw_0 C}{k_c^2} \left( \frac{n \pi}{b} \right) \cos (\frac{m \pi x}{a}) \sin (\frac{n \pi y}{b}) \sin (\frac{\beta d}{d}) \]

\[ E_y = -\frac{jw_0 C}{k_c^2} \left( \frac{m \pi}{a} \right) \sin (\frac{m \pi x}{a}) \cos (\frac{n \pi y}{b}) \sin (\frac{\beta d}{d}) \]

\[ \left( \beta = \frac{\pi}{a}, \quad k_c^2 = \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2 \right) \]

Resonant frequency: \( f_0 = \frac{1}{2 \pi \sqrt{2}} \left[ \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2 + \left( \frac{\beta d}{d} \right)^2 \right]^{\frac{1}{2}} \)
\[ E_x = \frac{D}{k_e} \left( \frac{\pi}{d} \right) \left( \frac{\pi}{a} \right) \sin \left( \frac{m\pi x}{a} \right) \cos \left( \frac{n\pi y}{b} \right) \cos \left( \frac{p\pi z}{d} \right) \]

\[ E_y = -\frac{D}{k_e} \left( \frac{\pi}{d} \right) \left( \frac{\pi}{b} \right) \sin \left( \frac{m\pi x}{a} \right) \cos \left( \frac{n\pi y}{b} \right) \sin \left( \frac{p\pi z}{d} \right) \]

\[ H_x = \frac{j \omega E}{k_e^2} \left( \frac{\pi}{b} \right) \sin \left( \frac{m\pi x}{a} \right) \cos \left( \frac{n\pi y}{b} \right) \cos \left( \frac{p\pi z}{d} \right) \]

\[ H_y = -\frac{j \omega E}{k_e^2} \left( \frac{\pi}{a} \right) \cos \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right) \cos \left( \frac{p\pi z}{d} \right) \]

**TM**<sub>mp</sub> and **TE**<sub>mp</sub> have identical resonant frequencies and different field patterns as degenerate modes. Additional degeneracies are generated by special combinations of dimensions (e.g., a = b = d \( \Rightarrow \) **TE**<sub>111</sub>, **TE**<sub>111</sub>, **TE**<sub>111</sub> become degenerate).

Resonant frequencies for higher-order modes get higher. To be resonant at a given frequency and a higher-order mode, the resonator has to be made bigger.
Q increases at a given frequency as one goes to higher order modes. Larger box has a greater volume-to-surface ratio, energy is stored in the volume and is lost on the imperfectly conducting surface.

**Resonator Perturbations**

At resonance, average stored magnetic and electric energies are equal. If a small perturbation is made on one cavity wall, this will in general change one type of energy more than the other. A shift in resonant frequency to again equalize energies.

When a small volume $\Delta V$ is removed:

$$\frac{\Delta W}{W_0} = \frac{\int_V (\mu H^2 - \epsilon E^2) \, dV}{\int_V (\mu H^2 + \epsilon E^2) \, dV} = \frac{\Delta U_M - \Delta U_E}{U}$$

(§) Perturbation of Resonant Parallel Plane Line

(A) Decrease length by $\Delta l$

```
<table>
<thead>
<tr>
<th>l - l'</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
</tr>
<tr>
<td>l'</td>
</tr>
</tbody>
</table>
```

$w$: width of line
Unperturbed resonance:

\[ l = \lambda/2 \quad \text{for lowest mode} \]

\[ \omega_0 = \frac{2\pi v_p}{A} = \frac{\pi v_p}{l} \]

Moving one plate in by \( \Delta l \), the new resonance is:

\[ (\omega_0 + \Delta\omega) = \frac{\pi v_p}{l-\Delta l} \approx \omega_0 (1 + \frac{\Delta l}{l}) \]

\[ \text{only magnetic energy is removed} \]

Assume unperturbed magnetic field

\[ H_0(z) = H_0 \sin \frac{\pi z}{l} \]

Total stored energy (twice average energy in H fields)

\[ U = 2 \pi d \int_0^{l/2} \frac{\mu H_0^2}{q} \sin^2 \frac{\pi z}{l} \, dz = \pi d \frac{\mu H_0^2}{q} \]

Energy removed:

\[ \Delta U_H = \frac{\pi d \mu H_0^2}{q} \]

\[ \omega = \frac{\Delta \omega}{\omega_0} = \frac{\Delta l}{l} \quad (\text{Equiv: Add an inductor in parallel}) \]

(b) Weak dent at the center (Remove only E-field)

Unperturbed E-field

\[ E_0(z) = E_0 \cos \frac{\pi z}{l} \]
Total energy stored:

\[ U = 2U_E = 2w d \int_{-a}^{a} \frac{\varepsilon E_0^2}{\varepsilon_0} \cos^2 \frac{\pi z}{L} \, dz \]

\[ = w L d \frac{\varepsilon E_0^2}{\varepsilon_0} \]

Electric energy removed:

\[ \Delta U_E = W \, d \Delta z \cdot \frac{\varepsilon E_0^2}{\varepsilon_0} \]

\[ \frac{\Delta w}{w_0} = - \frac{d \Delta z}{d z} \]

(Equiv. Add a capacitor in series)

\[ C_{new} = \frac{C_{conventional} \, C_{added}}{C_{conventional} + C_{added}} \]
COUPLING TO CAVITIES

1. Introduction of conducting probe or antenna in the direction of E-field lines driven by external transmission line.

2. Introduction of conducting loop with plane normal to the H-field lines. (Introduce inductive impedance)

3. Introduction of hole or iris between the cavity and a driving waveguide, the hole being located so that some field component in the cavity mode has a direction common to one in the wave mode.

4. Introduction of a pulsating electron beam passing through a small gap in the resonator, in the direction of E-field lines. (cyclotron type)
Microwave Networks

(A) Solve Maxwell's equations subject to boundary conditions for the entire system at once (very complicated and full of redundant information)

(B) Circuit approach that describes the characteristics of each part of the system as a transducer or power-pleasant transfer element between units or coupling element to adjacent units.

Certain waveguide or transmission line inputs and outputs

For waveguides, we wish to create only the dominant mode. The cutoff (evanescent) modes (close to discontinuities) are introduced as reactive elements.

Assume linear and isotropic medium.

NETWORK FORMULATION

For TEM mode, V, I are defined in a unique way.

For non-TEM waves,
(1) V = Transverse E field of the mode
(2) I = Transverse H field of the mode
Average power is $\text{Re} \{ |V|/2 \}^2$.

(3) $\frac{V}{I}$ of an incident wave is characteristic impedance of the guide.

(Usually $z$ for normalization)

Ex. TE$_{10}$ in rectangular guide

\[
\begin{align*}
E_t(x,y,z) &= V_0 \, e^{-y^2} \, F(x,y) \\
H_t(x,y,z) &= I_0 \, e^{-x^2} \, G(x,y)
\end{align*}
\]

\[
\frac{W_t}{V_0} = \text{Re} \left \{ V_0 I_0 \right \}
\]

\[
\frac{V_0}{I_0} = \frac{z_0}{\lambda_0}
\]

$W_t = V_0 I_0^* = 2b \int_0^a \frac{E_0^2}{2z_0^2} \sin^2 \frac{\pi x}{a} \, dx = \frac{E_0^2}{2 z_0^2} \frac{a^2}{\pi^2}$

\[
V_0 = E_0 \left( \frac{ab z_0}{2 z_0} \right)^{1/2}, \quad I_0 = \left( \frac{E_0}{z_0} \right) \left( \frac{ba z_0}{2 z_0} \right)^{1/2}
\]

\[
F(x) = \left( \frac{2 z_0}{ab z_0} \right)^{1/2} \sin \left( \frac{\pi x}{a} \right), \quad G(x) = - \left( \frac{2 z_0}{ba z_0} \right)^{1/2} \sin \left( \frac{\pi x}{b} \right)
\]
For linear media:

\[ I_1 = Y_{11} V_1 + Y_{12} V_2 + Y_{13} V_3 \]
\[ I_2 = Y_{21} V_1 + Y_{22} V_2 + Y_{23} V_3 \]
\[ I_3 = Y_{31} V_1 + Y_{32} V_2 + Y_{33} V_3 \]

and

\[ V_1 = Z_{11} I_1 + Z_{12} I_2 + Z_{13} I_3 \]
\[ V_2 = Z_{21} I_1 + Z_{22} I_2 + Z_{23} I_3 \]
\[ V_3 = Z_{31} I_1 + Z_{32} I_2 + Z_{33} I_3 \]
Conditions for Reciprocity

\[ Y_{ij} = Y_{ji}, \quad Z_{ij} = Z_{ji} \]

Symmetrical matrix around diagonal

(A) Circuit Definition

\[ \begin{align*}
Y_{11} & = \frac{Y}{2} \\
Y_{12} & = \frac{Z}{2}
\end{align*} \]

Reciprocal Network

and Circulate (Non-reciprocal separate transmission & reception)

(B) Field Definition

\[ \begin{align*}
\vec{E}_a, \vec{H}_a & \rightarrow \begin{bmatrix} \vec{E}_1 \, \vec{H}_1 \end{bmatrix} \\
\vec{E}_b, \vec{H}_b & \rightarrow \begin{bmatrix} \vec{E}_2 \, \vec{H}_2 \end{bmatrix}
\end{align*} \]

\[ \begin{align*}
\vec{E}_a \times \vec{H}_b - \vec{E}_b \times \vec{H}_a & = 0
\end{align*} \]

Always satisfied for isotropic (but necessarily homogenous) media. For anisotropic media, it holds provided the permittivity and permeability matrices are symmetric.

If \([\varepsilon]\) or \([\mu]\) are asymmetric = non-reciprocal (gyrotropic) media.

Two-Port Waveguide Junction

(Filter, WG transition, Phase-converter devices)
The **ABCD matrix** correlates input quantities in terms of output ones:

\[
\begin{bmatrix}
I_1 \\
I_2 \\
J_1 \\
J_2
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
\]

\[Y_{11} = \frac{Z_{22}}{\Delta (\tau)} = \frac{D}{B}\]

\[Y_{12} = -\frac{Z_{12}}{\Delta (\tau)} = -\frac{(AD-BC)}{B}\]

\[Y_{21} = -\frac{Z_{21}}{\Delta (\tau)} = -\frac{1}{B}\]

\[Y_{22} = \frac{Z_{11}}{\Delta (\tau)} = \frac{A}{B}\]

with: \[\Delta (\tau) = Z_{11}Z_{22} - Z_{12}Z_{21}\]

**Reciprocal network**: \[Z_{11} = Z_{12}, \quad Y_{11} = Y_{12}, \quad AD-BC = 1\]

(Assumed for the remainder of y-wave networks)
\[ \tan(\beta_1 l_1) = \frac{1 + c^2 - a^2 - b^2}{2 (bc - a)} + \sqrt{\frac{1 + c^2 - a^2 - b^2}{2 (bc - a)}} \]

\[ \alpha = -j \frac{Z_{11}}{Z_{21}} \]

\[ b = \frac{Z_{11} Z_{22} - Z_{12}^2}{Z_{21} Z_{22}} \]

\[ c = -j \frac{Z_{12}}{Z_{22}} \]
Expressed in terms of reflected/incident waves is more useful than total voltages/ currents.

Normalization: \[ a_n = \frac{V_n^+}{\sqrt{Z_n}}, \quad b_n = \frac{V_n^-}{\sqrt{Z_n}} \]

At reference plane \( n \):
\[ V_n = V_n^+ + V_n^- = \sqrt{Z_n} \left( a_n + b_n \right) \]
\[ I_n = \frac{1}{Z_n} \left( V_n^+ - V_n^- \right) = \frac{1}{\sqrt{Z_n}} \left( a_n - b_n \right) \]

Average power flowing into terminal \( n \):
\[
(W_n)_{av} = \frac{1}{2} \text{Re} \left( V_n I_n^* \right) = \frac{1}{2} \text{Re} \left[ (a_n a_n^* - b_n b_n^*) + (b_n a_n^* - b_n a_n^*) \right]
\]

\[
\begin{bmatrix}
2(W_n)_{av} = a_n a_n^* - b_n b_n^*
\end{bmatrix}
\]

\[ b_1 = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \]

\[ b_3 = [C] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \]

incident reflected scattering matrix
\[ b_1 = S_{11} a_1, \quad b_2 = S_{21} a_1 \]

\[ C: \text{Source applied to port 1, Output guide matched to } Z_{22} \]

\[ \text{Input: } 2 (W, \omega) a_1 = (1 - S_{11} S_{21}) a_1 a_1^* \]

\[ \text{Output: } 2 (W, \omega) a_1 = -S_{21} S_{21}^* a_1 a_1^* \]

(Power power toward the port)

Passive network: \( S_{21} S_{21}^* \leq 1 - S_{11} S_{11}^* \)

\[ F S_{11} = (Z_{11} - Z_{21}) (Z_{11} + Z_{22}) - Z_{12} Z_{21} \]

\[ F S_{12} = 2 \sqrt{Z_{21} Z_{22}} Z_{12} \]

\[ F S_{21} = 2 \sqrt{Z_{21} Z_{22}} Z_{21} \]

\[ F S_{22} = (Z_{11} - Z_{22}) (Z_{11} + Z_{21}) - Z_{21} Z_{12} \]

\[ F = (Z_{11} + Z_{21}) (Z_{12} + Z_{22}) - Z_{12} Z_{21} \]

Reciprocity: \( S_{21} = S_{12} \)

\( T \) (Transmission Matrix)

\[ \begin{align*}
\{ b_2 &= T_{11} a_1 + T_{12} b_1 \\
a_1 &= T_{21} a_1 + T_{22} b_1 \end{align*} \]

\[ T_{11} = S_{21} * - \frac{S_{11} S_{22}}{S_{12}}, \quad T_{12} = \frac{S_{12}}{S_{11}} \]

\[ T_{21} = -\frac{S_{11}}{S_{12}}, \quad T_{22} = \frac{1}{S_{12}} \]