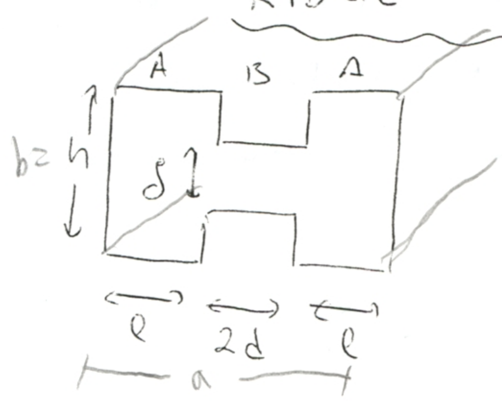
ECE 3065

LECTURE 22

RIDGE WAVEGUIDE



The cutoff frequency is lowered because of the capacitive effect at the center. Can be further decreased as $\delta \downarrow$. Applied to nonuniform tr. lines for matching purposes.

At ~~resonance~~ cutoff, there is no variation to z-direction ($\gamma = 0$) $\Leftrightarrow$ resonant condition for waves propagating only transversely in the given cross section.

Approximation for TE_{10} (dependence only on x direction)

Gap $\leftrightarrow$ Capacitance

$$f_c = \frac{1}{2\pi\sqrt{LC}}$$

Side sections $\leftrightarrow$ One-turn solenoidal inductances

$$f_c = \frac{1}{2\pi}\left(C_B \frac{L_A}{2}\right)^{-1/2} = \frac{1}{2\pi}\left(\frac{2d\varepsilon}{\delta}\right)^{-1/2}\left(\mu\frac{lh}{2}\right)^{-1/2}$$

$$= \frac{1}{2\pi}\left(\frac{\delta}{\mu\varepsilon l h d}\right)^{1/2}$$

Good approximation for smaller gaps.

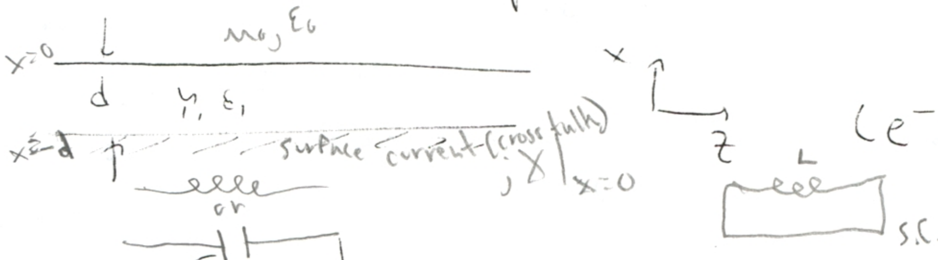
(Transverse Resonance Frequency Technique)

(-) Decreased power-handling capability

(+) Smaller dimensions than rectangular WG

→ SURFACE GUIDING

A surface that possesses a reactive surface impedance keeps the fields localized on it ⇝ surface-guiding ⇝ the energy is maintained near the surface and it is not radiated or coupled seriously to nearby objects



$(e^{-j\beta z})$

TM waves
↓
between $x=-d, 0$

$$E_z = 0 \Big|_{PEC\,(x=-d)} = D \sin k_x (x+d)$$

$$H_y = -\frac{j\omega\varepsilon_1 D}{k_x} \cos k_x (x+d)$$

$$E_x = -\frac{j\beta D}{k_x} \cos k_x (x+d)$$

$$k_x^2 = k_1^2 - \beta^2 \qquad (k_1 = \omega\sqrt{\mu_1\varepsilon_1})$$

Field impedance at $x=0$ -

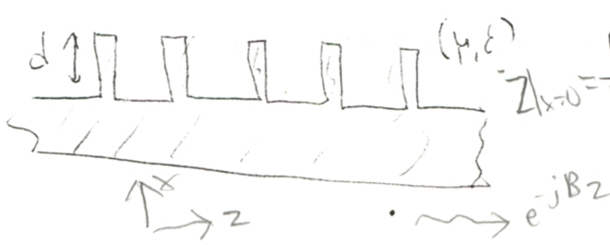
$$jX = \frac{E_z}{H_y}\Big|_{x=0} = \frac{jk_x}{\omega\varepsilon_1} \tan k_x d \overset{k_x d \ll 1}{\approx} \frac{jk_x^2 d}{\omega\varepsilon_1}$$

imaginary
Inductive

inductor

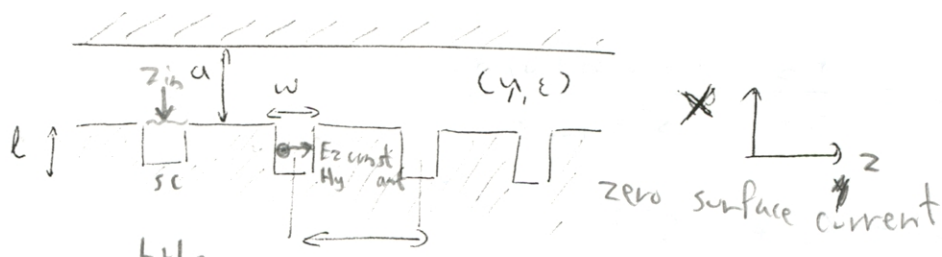
On surface guided waves the phase velocity is less than the velocity of light in the external dielectric and greater than that of the coating
μ_0, ε_0

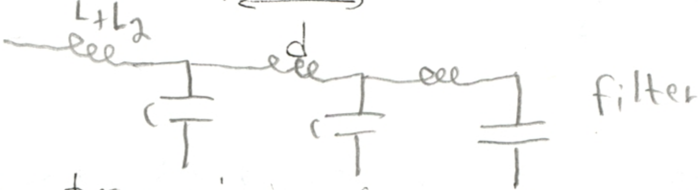
- Similarly:



$$Z|_{x=0} = \frac{E_z}{H_y} = j\eta \tan kd, \quad k = \omega\sqrt{\mu\varepsilon}, \quad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

~~k=~~





$C = \varepsilon \frac{d-w}{a}$

$L = \mu a (d-w)$

$Z_{in}(sc)|_\ell = j \frac{\eta w}{w} \tan k\ell \approx L_2$

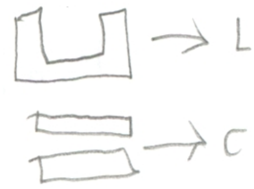
Parallel-plane transmission line with one-side periodic troughs.

Narrow troughs → disturb bottom z-directed currents
→ introduce shorts in series with bottom conductors.

↓

Constant E_z, H_y across the gap width w

$\frac{\partial}{\partial y} = 0$. infinitely long Consider waves E_x, E_z, H_y (TM) $H_z = 0$



Satisfy. $E_z = 0 |_{x=a}$ periodic structure w/ period d

Assume summations of TM waves ($n=0 \to$ TEM)

$$E_z(x, z) = \sum_n A_n \sin K_n (a-x) e^{-j\beta_n z}$$

$$E_x(x, z) = \sum_n \frac{j\beta_n}{K_n} A_n \cos K_n (a-x) e^{-j\beta_n z}$$

$$H_y(x, z) = \sum_n \frac{j\omega\varepsilon}{K_n} A_n \cos K_n (a-x) e^{-j\beta_n z}$$

$$K_n^2 = \omega^2 \mu\varepsilon - \beta_n^2 = k^2 - \beta_n^2$$

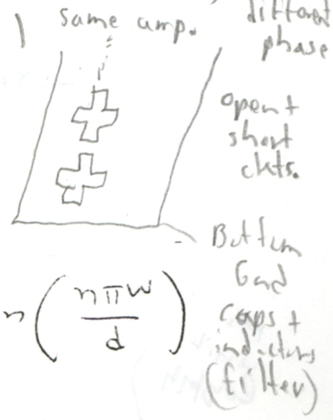
Floquet Thrm.

$E_z(x, z = md) = E_z(x, z=0) e^{-j\beta_0 md}$

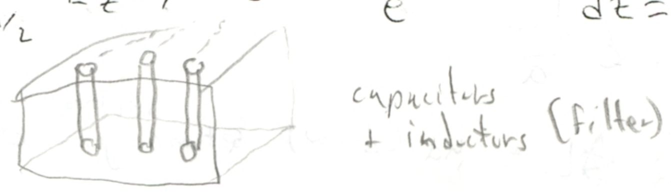
same amp. different phase

$E_z(0, z)$ is a periodic function of z analyze for 1 period

$$\sim \quad E_z(0, z) = e^{-j\beta_0 z} \sum_n C_n e^{-j 2\pi n z/d}$$



$$C_n = \frac{1}{d} \int_{-d/2}^{d/2} E_z(0, z) e^{j\beta_0 z} e^{j 2\pi n z/d} dz = \frac{E_0}{\pi n} \sin\left(\frac{n\pi w}{d}\right)$$



$$\Rightarrow \begin{cases} A_n = \dfrac{E_0}{\pi n} \dfrac{\sin(n\pi w/d)}{\sin K_n a} \\ \beta_n = \beta_0 + \dfrac{2\pi n}{d} \end{cases}$$

Here: TM modes are coupled by the periodic boundary condition and have to coexist!!
("spatial harmonics")

Assume that the trough is approximated by a short:

$$H_{y0}\big|_{x=0,\,z=0} = -\frac{jE_0}{\eta} \cot kl$$

$$\Downarrow$$

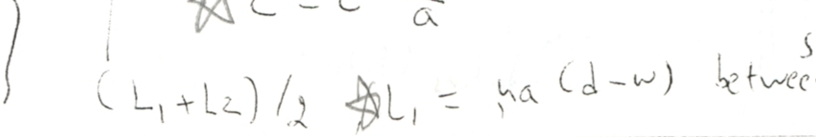
$$-\frac{j}{\eta} \cot kl = \sum \frac{j\omega\varepsilon}{\pi n K_n} \sin \frac{\pi n w}{d} \cot K_n a$$

$$\begin{cases} K_n^2 = k^2 - \beta_n^2 \\ \beta_n = \beta_0 + \dfrac{2\pi n}{d} \end{cases}$$

$$\Downarrow$$

$$\beta_0 = \ldots$$

✩ Lumped equiv. circuit ($\beta_0 d \ll 1$)



per unit length

✩ $C = \varepsilon \dfrac{d-w}{a}$

$(L_1 + L_2)/2$ ✩ $L_1 = \mu a (d-w)$ between section grooves, $L_2 = \dfrac{\eta w}{\omega} \tan kl$ (S.C.

(·) $\sum_n = \sum_{n=-\infty}^{-1} \cdots + \sum_{n=0}^{\infty} \cdots$

backward   forward

propagation

(·) Group velocities of all spatial harmonics is equal:

$$\frac{1}{v_{gn}} = \frac{d\beta_n}{d\omega} = \frac{d\beta_0}{d\omega}$$

(·) Floquet's theorem (Fundamental for the study of periodic structures)

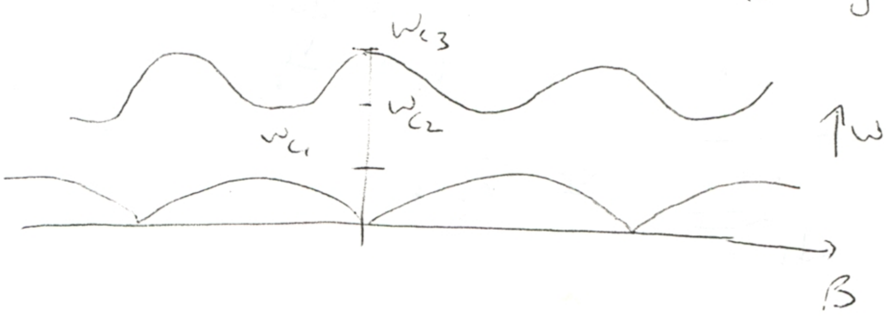
Field distribution at plane $z = md$ ($m \in N$) is the same as at $z = 0$, except from factor $e^{-j\beta_0 md}$

eg. $E_z(x, md) = e^{-j\beta_0 md} \, E_z(x, 0)$

(·) The wave is cut off where $v_g = 0$

Above this frequency $\Rightarrow$ reactive (filter) attenuation

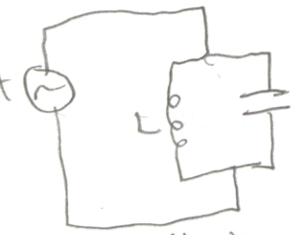
<u>BUT</u>: other pass bands exist at higher frequencies

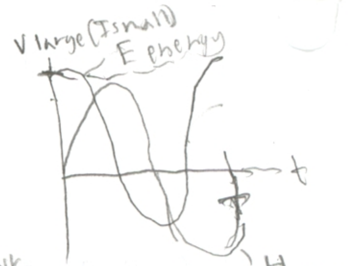


LECTURE 23

Resonant circuits

(a) Lumped-element type





Capacitor for storage of E-energy

Inductor for — " — of H-energy

At resonance there is exchange of energy.

can't handle large pwr.

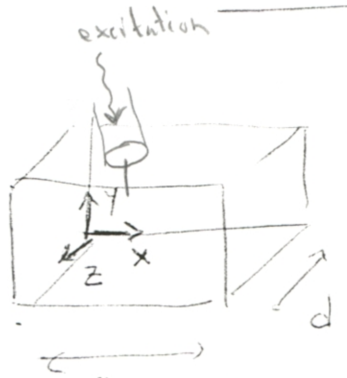
same amplitudes

(b) Distributed type

standing wave set up by interference between forward and backward traveling waves (Shorts)

Cavity - stores energy

RECTANGULAR CAVITY - handles more pwr.



TE_{101} dominant mode

Resonance: standing wave corresponding to TE_{10} of a rectangular guide

Assume only $E_y \rightarrow \hat{z}$

$E_y = 0 \Big|_{z=0,\, z=d}$ $\quad E_x = 0 \Big|_{z=0,d}$

$$d = \frac{\lambda_g}{2} = \frac{\lambda}{2\sqrt{1-(\lambda/2a)^2}}$$

$$f_c^{TE_{10}} = \frac{v_p}{2}\frac{1}{a}$$

Resonant frequency: $v_p = \frac{1}{\sqrt{\mu\varepsilon}}$

$$f_0^{TE_{101}} = \frac{v}{\lambda} = \frac{\sqrt{a^2+d^2}}{2ad\sqrt{\mu\varepsilon}} = \frac{v_p}{2}\sqrt{\frac{a^2+d^2}{(ad)^2}} = \frac{v_p}{2}\sqrt{\frac{1}{d^2}+\frac{1}{a^2}} > f_c\ TE_{10}$$

$$TE_{10}\begin{cases} E_y = \left(E_+ e^{-j\beta z} + E_- e^{j\beta z}\right)\sin\frac{\pi x}{a} \\ H_x = -\frac{1}{Z_{TE}}\left(E_+ e^{-j\beta z} - E_- e^{j\beta z}\right)\sin\frac{\pi x}{a} \end{cases}$$

$$H_z = \frac{j}{\eta}\left(\frac{\lambda}{2a}\right)\left(E_+ e^{-j\beta z} + E_- e^{j\beta z}\right)\cos\frac{\pi x}{a}$$

$$(E_+ + E_-)\sin\left(\frac{\pi x}{a}\right) = 0$$

$$E_y = 0\,\big|_{z=0} \Rightarrow E_- = -E_+$$

$$E_y = E_+\underbrace{\left(e^{-j\beta z} - e^{j\beta z}\right)}_{(-2j\sin\beta z)}\sin\left(\frac{\pi x}{a}\right) = E_0 \sin(\beta z)\sin\left(\frac{\pi x}{a}\right)$$

$$E_0 = -2jE_+$$

$$E_y = 0\,\big|_{z=d} \Rightarrow \beta = m\pi/d \qquad \sin(\beta d) = 0, \quad m = 1,2,3\ldots$$

$$E_y = E_0 \sin\frac{\pi x}{a}\sin\frac{m\pi z}{d} \qquad TE_{10m}$$

$$H_x = -j\frac{E_0}{\eta}\frac{\lambda}{2d}\sin\frac{\pi x}{a}\cos\frac{\pi z}{d}$$

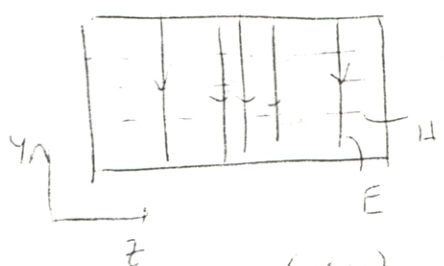
$$H_z = j\frac{E_0}{\eta}\frac{\lambda}{2a}\cos\frac{\pi x}{a}\sin\frac{\pi z}{d}$$

Dominant mode TE_{101} mode

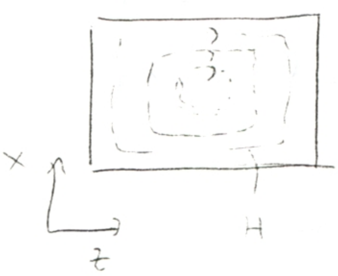
x y z

2 1's

Field patterns $E_x = 0 = H_y = E_z$



(x y)



The total energy passes between electric and magnetic fields.

= electric field when E-field is max

$$U = (U_E)_{max} = \frac{\varepsilon}{2}\int_0^d\int_0^b\int_0^a |E_y|^2\,dx\,dy\,dz = \frac{\varepsilon abd}{8}E_0^2 = \frac{\varepsilon}{8}E_0^2 \cdot \text{Volume}$$

$$\frac{1}{2}\iiint_V \varepsilon |E^2|\,dV = \frac{\varepsilon}{2}\,\frac{a}{2}\,b\,\frac{d}{2}\,E_0^2$$

Volume↑ → U↑

$\int \sin^2 x = \frac{1}{2}$

Power loss estimation $J = \hat{n}\times\vec{H}$

Front: $J_{sy} = -H_x|_{z=d}$, Back: $J_{sy} = H_x|_{z=0}$

Left: $J_{sy} = -H_z|_{x=0}$, Right: $J_{sy} = H_z|_{x=a}$

Top: $J_{sx} = -H_z$, $J_{sz} = H_x$, Bottom: $J_{sx} = H_z$, $J_{sz} = -H_x$

if the conducting walls have resistivity R_s: $P = \frac{1}{2}RI^2$

$$\text{loss} = W_L = \frac{R_s}{2}\Big\{2\underbrace{\int_0^b\int_0^a |H_x|^2_{z=0}\,dx\,dy}_{\text{front + back}} + 2\underbrace{\int_0^d\int_0^b |H_z|^2_{x=0}\,dy\,dz}_{\text{sides}} + 2\underbrace{\int_0^d\int_0^a \left(|H_x|^2 + |H_z|^2\right)dx\,dz}_{\text{top + bottom}}\Big\}$$

$$W_L = \frac{R_S\, \lambda^2}{8\eta^2} E_0^2 \left[\frac{ab}{d^2} + \frac{bd}{a^2} + \frac{1}{2}\left(\frac{a}{d} + \frac{d}{a}\right)\right]$$

depends on surface areas

$$\boxed{Q = \frac{\omega_0 U}{W_L}}$$

most crit. #
quality factor Q ↑ good

$\Downarrow \sim \frac{\omega_0 \text{ Volume}}{\text{surface area}}$

$$Q = \frac{\pi \eta}{4 R_S} \left[\frac{2b\,(a^2+d^2)^{3/2}}{ad\,(a^2+d^2) + 2b\,(a^3+d^3)} \right]$$

Use fresnant.

For cube: $a = b = d \Rightarrow Q = 0.742 \frac{\eta}{R_S}$

Usually very large Q in comparison to lumped resonators (~ 100's)

Dielectric losses and radiation from small holes lower Q !!

Bandwidth of a resonator

$$\frac{\Delta f}{f_0} \approx \frac{1}{Q}$$

Δf: distance between points on the response curve for which amplitude response is down to $\frac{1}{\sqrt{2}}$ of its maximum value ("half-power" points)

LECTURE 24

HIGHER ORDER RESONATOR MODES

For a rectangular waveguide, the TE_{mn} mode is:

$$H_z = (Ae^{-j\beta z} + Be^{j\beta z}) \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}$$

In the resonators: $H_z \equiv 0 \big|_{z=0,\,d}$

$$\rightsquigarrow B = -A, \quad \beta d = p\pi \quad , p \in N$$

$$\Downarrow$$

$$H_z = A(e^{-j\beta z} - e^{j\beta z}) \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} =$$

$$= -A\, 2j \sin(\beta z) \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} =$$

$$\overset{C=2jA(-1)}{=\!=\!=} C \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right)$$

$$H_x = -\frac{C}{k_c^2}\left(\frac{p\pi}{d}\right)\left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \cos\left(\frac{p\pi z}{d}\right)$$

$$H_y = -\frac{C}{k_c^2}\left(\frac{p\pi}{d}\right)\left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \cos\left(\frac{p\pi z}{d}\right)$$

$$E_x = \frac{j\omega\mu C}{k_c^2}\left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right)$$

$$E_y = -\frac{j\omega\mu C}{k_c^2}\left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right)$$

$$\left(\beta = \frac{p\pi}{d} \quad , k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)$$

Resonant frequency: $f_0 = \dfrac{1}{2\pi\sqrt{\mu\varepsilon}}\left[\left(\dfrac{m\pi}{a}\right)^2 + \left(\dfrac{n\pi}{b}\right)^2 + \left(\dfrac{p\pi}{d}\right)^2\right]^{1/2}$

TM_{mnp} Modes

$$E_z = D \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \cos\left(\frac{p\pi z}{d}\right)$$

$$E_x = -\frac{D}{k_c^2}\left(\frac{p\pi}{d}\right)\left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right)$$

$$E_y = -\frac{D}{k_c^2}\left(\frac{p\pi}{d}\right)\left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right)$$

$$H_x = \frac{j\omega\varepsilon D}{k_c^2}\left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \cos\left(\frac{p\pi z}{d}\right)$$

$$H_y = -\frac{j\omega\varepsilon D}{k_c^2}\left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \cos\left(\frac{p\pi z}{d}\right)$$

TE_{mnp}, TM_{mnp} have identical resonant frequencies and different field patterns $\sim$ degenerate modes. Additional degeneracies are generated by special combinations of dimensions (cf. $a=b=d \sim TE_{112}, TE_{211}, TE_{121}$ become degenerate)

Resonant frequency for higher-order modes gets higher. To be resonant at a given frequency and a higher order mode, the resonator has to be made bigger

Q increases at a given frequency as one goes to higher order modes.

(Larger box has a greater volume-to-surface ratio, energy is stored in the volume and is lost on the imperfectly conducting surface)

RESONATOR PERTURBATIONS

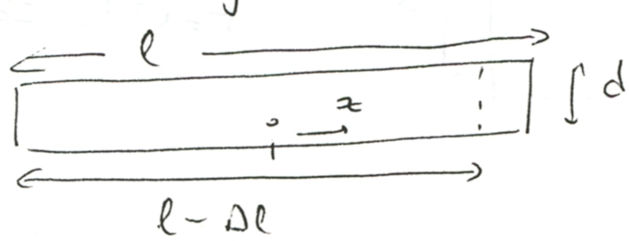
At resonance, average stored magnetic and electric energies are equal. If a small perturbation is made on one cavity wall, this will in general change one type of energy more than the other ~ shift in resonant frequency to again equalize energies.

When a small volume ΔV is removed:

$$\frac{\Delta \omega}{\omega_0} = \frac{\int_{\Delta V} (\mu H^2 - \varepsilon E^2)\, dV}{\int_V (\mu H^2 + \varepsilon E^2)\, dV} = \frac{\Delta U_H - \Delta U_E}{U}$$

(S) Perturbation of Resonant Parallel-Plane Line

(A) Decrease length by Δl



w : width of line

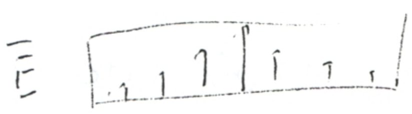
Unperturbed resonance:

$$l = \lambda/2 \quad \text{for lowest mode}$$

$$\omega_0 = \frac{2\pi v_p}{\lambda} = \frac{\pi v_p}{l}$$

Moving one plate in by Δl, the new resonance is:

$$(\omega_0 + \Delta\omega) = \frac{\pi v_p}{l - \Delta l} \approx \omega_0 \left(1 + \frac{\Delta l}{l}\right)$$



→ only magnetic energy is removed

Assume unperturbed magnetic field

$$H_0(z) = H_0 \sin \frac{\pi z}{l}$$

Total stored energy (twice average energy in H fields)

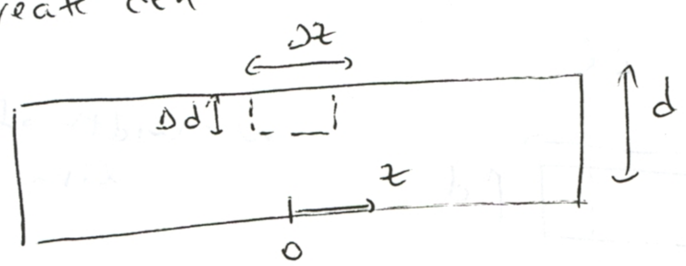
$$U = 2wd \int_{-l/2}^{l/2} \frac{\mu H_0^2}{4} \sin^2 \frac{\pi z}{l}\, dz = w l d \frac{\mu H_0^2}{4}$$

Energy removed: $\Delta U_H = \dfrac{\mu H_0^2}{4} w d \Delta l$

$$\Rightarrow \frac{\Delta\omega}{\omega_0} = \frac{\Delta l}{l}$$

(Equiv: Add an inductor in parallel)

(B) Create dent at the center (Remove only E-field)



Unperturbed E-field

$$E_0(z) = E_0 \cos \frac{\pi z}{l}$$

Total energy stored:

$$U = 2U_E = 2wd \int_{-\ell/2}^{\ell/2} \frac{\varepsilon E_0^2}{4} \cos^2 \frac{\pi z}{\ell}\, dz$$

$$= w \ell d \frac{\varepsilon E_0^2}{4}$$

El. energy removed:

$$\Delta U_E = W\, \Delta d\, \Delta z \cdot \frac{\varepsilon E_0^2}{4}$$

$$\Downarrow$$

$$\frac{\Delta \omega}{\omega_0} = - \frac{\Delta d \cdot \Delta z}{d \cdot z}$$

(Equiv: Add a capacitance in series)

$$C_{NEW} = \frac{C_{unperturbed}\, C_{Added}}{C_{unperturbed} + C_{Added}}$$

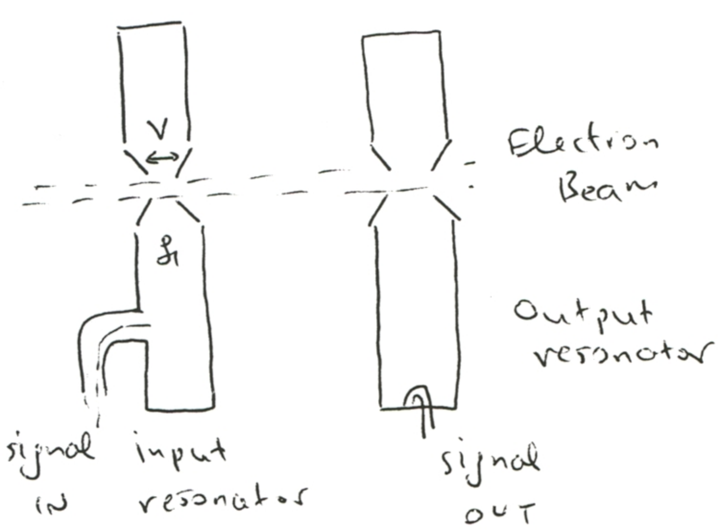
COUPLING TO CAVITIES

1. Introduction of conducting probe or antenna in the direction of $\bar{E}$-field lines, driven by external transmission line.

2. Introduction of conducting loop with plane normal to the $\bar{H}$-field lines. (Introduces inductive impedance)

3. Introduction of hole or iris between the cavity and a driving waveguide, the hole being located so that some field component in the cavity mode has a direction common to one in the wave mode.

4. Introduction of a pulsating electron beam passing through a small gap in the resonator, in the direction of $\bar{E}$-field lines. (klystron type)



LECTURE 25

Microwave Networks

(A) Solve Maxwell's equations subject to boundary conditions for the entire system at once (very complicated and full of redundant information)

(B) Circuit approach that describes the characteristics of each part of the system as a transducer or power-~~element~~ transfer element between units or coupling element to adjacent units.

↓

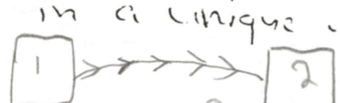
Certain waveguide or transmission-line inlets and outlets

For waveguides, we wish to excite only the dominant mode. The cutoff (evanescent) modes close to discontinuities are introduced as reactive elements!!

Assume linear and isotropic medium.

NETWORK FORMULATION

For TEM mode: V, I are defined in a unique way



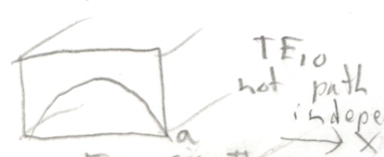
For non-TEM waves:

$V = \int E \cdot dL$ path independent for TEM

$I = \int H \cdot dL$

(1) V ~ Transverse $\bar{E}$ field of the mode

I ~ Transverse $\bar{H}$ field of the mode



TE_{10} not path independent

$E \sim \sin \frac{\pi}{a} x$

(2) Average power $= \mathrm{Re}[VI^*/2]$

(3) $\frac{V}{I}$ of an incident wave = charact impedance of the mode

(Usually = 1 for normalization)

eg. TE_{10} for rectangular guide

$$\bar{E}_t(x,y,z) = V_0 e^{-\gamma z} \bar{f}(x,y)$$

$$\bar{H}_t(x,y,z) = I_0 e^{-\gamma z} \bar{g}(x,y)$$

transferred energy

$$W_T = \mathrm{Re}(V_0 I_0^*)$$

$$\frac{V_0}{I_0} = Z_0$$

$\bar{f}(x,y)$

F_y

$$\bar{E}_t(x,y,z) = \left(\sin\left(\frac{\pi x}{a}\right)\right)\hat{y}\, e^{-\gamma z} V_0$$

$$E_y = E_0 \sin\frac{\pi x}{a} = V_0 f(x)$$

$$H_x = -\frac{E_0}{Z_{zTE_{10}}} \sin\frac{\pi x}{a} = I_0 g(x)$$

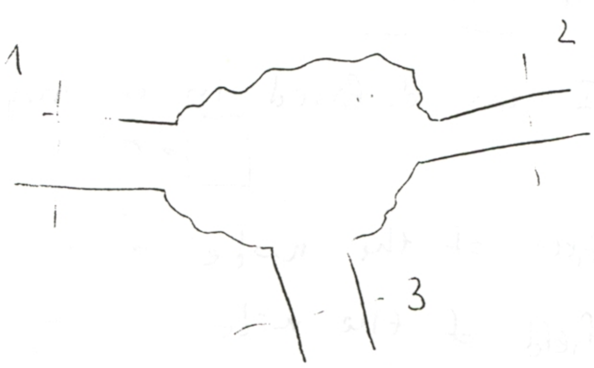
$$\bar{H}_t(x,y,z) = \underbrace{\left(\sin\frac{\pi x}{a}\right)\hat{x}}_{g(x,y)}\, e^{-\gamma z} I_0$$

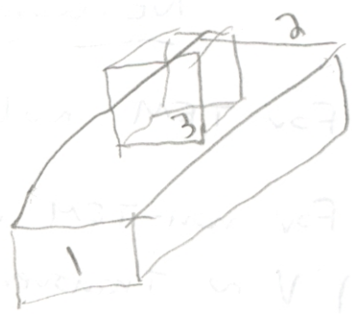
$$W_T = V_0 I_0^* = 2b\int_0^a \frac{E_0^2}{2Z_z}\sin^2\frac{\pi x}{a}\,dx = \frac{ab\,E_0^2}{2Z_{zTE_{10}}}$$

$$\frac{V_0}{I_0} = Z_0$$

$$V_0 = E_0\left(\frac{ab Z_0}{2 Z_z}\right)^{1/2}, \quad I_0 = \left(\frac{E_0}{Z_z}\right)\left(\frac{ba Z_z}{2Z_0}\right)^{1/2}$$

$$\left(f(x) = \left(\frac{2Z_z}{ab Z_0}\right)^{1/2} \sin\left(\frac{\pi x}{a}\right), \quad g(x) = -\left(\frac{2Z_0}{ba Z_z}\right)^{1/2} \sin\left(\frac{\pi x}{a}\right) \right)$$





For linear media:

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3$$
$$I_3 = Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3$$

$\hookrightarrow [I] = [Y][V]$

Y_{xy} — x: current port, y: voltage port

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \underbrace{\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}}_{\text{Y-matrix}} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

and

$$V_1 = Z_{11}I_1 + Z_{12}I_2 + Z_{13}I_3$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 + Z_{23}I_3$$
$$V_3 = Z_{31}I_1 + Z_{32}I_2 + Z_{33}I_3$$

$\hookrightarrow [V] = [Z][I]$

Z-matrix

Z_{xy} — x: voltage port, y: current port

LECTURE 26

C:

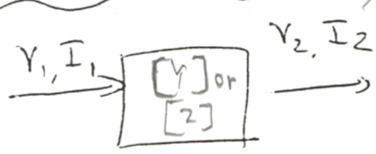
Conditions for Reciprocity

$Y_{ij} = Y_{ji}$, $Z_{ij} = Z_{ji}$ symmetrical matrix around diagonal

$Y_{12} = Y_{21}$ same $\left(\begin{array}{l} I_1 = f(V_2) \\ I_2 = f(V_1) \end{array} \right.$

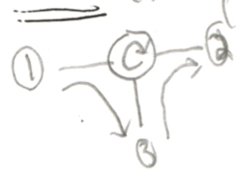
(A) Circuit Definition

Reciprocal Network



and

Circulator (Non-reciprocal)



separate transmission + reception

(B) Field Definition

and

same $\left(\begin{array}{l} H_a = f(E_b) \rightarrow E_b \times H_a \\ H_b = f(E_a) \rightarrow E_a \times H_b \end{array} \right.$ or

$$\nabla \cdot (\tilde{E}_a \times \tilde{H}_b - \tilde{E}_b \times \tilde{H}_a) = 0$$

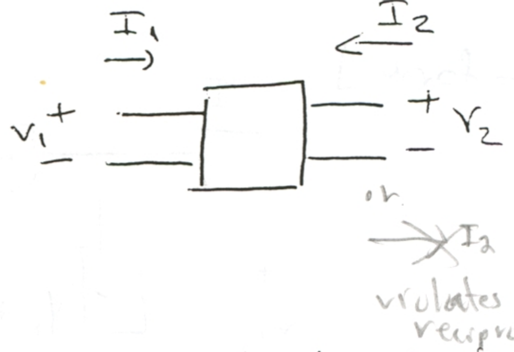
Always satisfied for isotropic (not necessarily homogeneous) media. For anisotropic media, it holds provided the permittivity and permeability matrices are symmetric.
If $[\varepsilon]$ or $[\mu]$ are assymetric $\Rightarrow$ non-reciprocal (gyrotropic) media

Two-Port Waveguide Junction

(Filters, WG transitions, Phase-correction devices)

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



→ ABCD matrix : correlates input quantities in terms of output one

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Relationship [Y], [Z], [ABCD]

$$Y_{11} = \frac{Z_{22}}{\Delta(Z)} = \frac{D}{B}$$

$$Y_{12} = \frac{-Z_{12}}{\Delta(Z)} = \frac{-(AD-BC)}{B}$$

$$Y_{21} = \frac{-Z_{21}}{\Delta(Z)} = -\frac{1}{B}$$

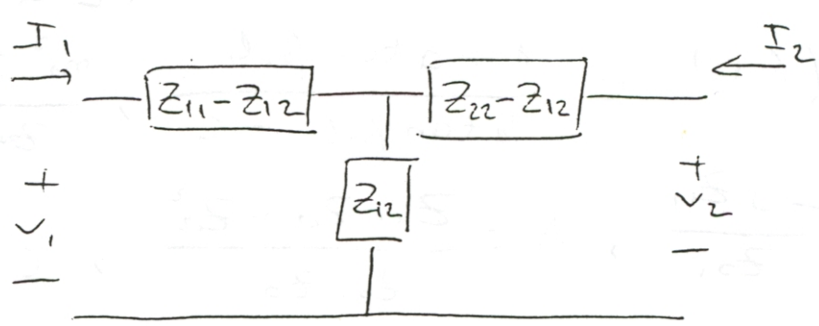
$$Y_{22} = \frac{Z_{11}}{\Delta(Z)} = \frac{A}{B}$$

with: $\Delta(Z) = Z_{11} Z_{22} - Z_{12} Z_{21}$

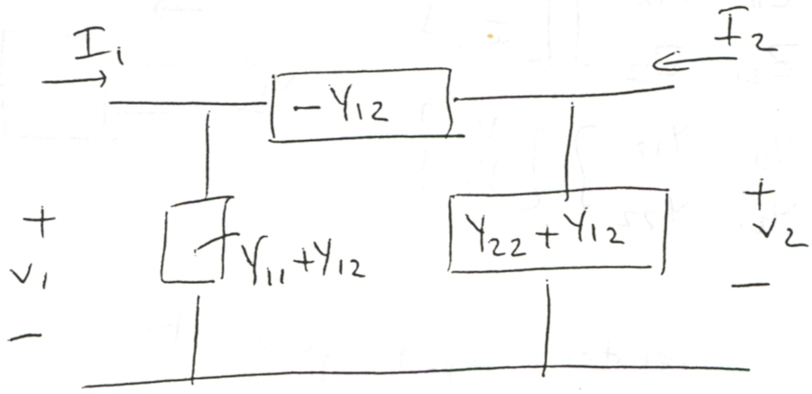
Reciprocal Network: $Z_{21} = Z_{12}$, $Y_{21} = Y_{12}$, $AD - BC = 1$

(Assumed for the remainder of μwave networks)

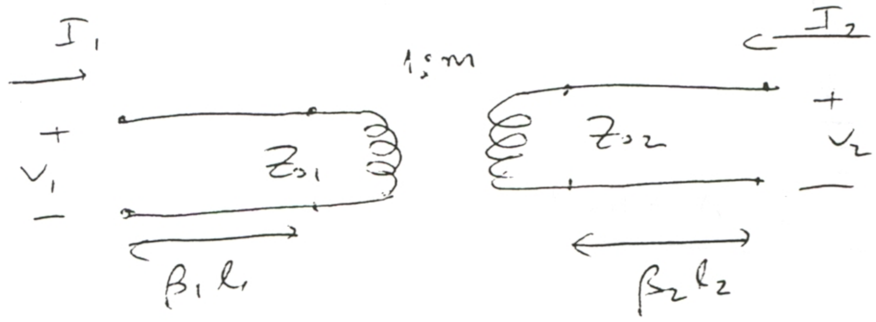
[T - form]



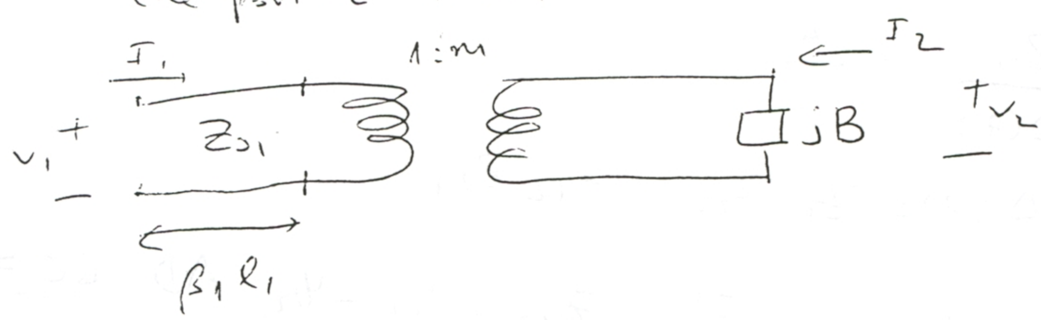
[π - form]



[Ideal transformer with sections of transmission lines
(account for arbitrary reference planes)]



[Alternative form with shunt element instead of the port-2 transmission line]



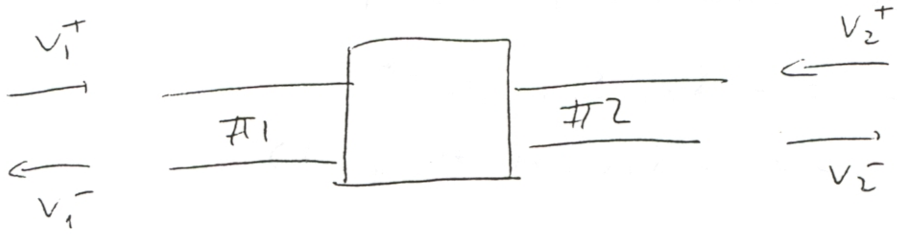
$$\tan(\beta_1 l_1) = \left[\frac{1+c^2-a^2-b^2}{2(bc-a)} \pm \sqrt{\left[\frac{1+c^2-a^2-b^2}{2(bc-a)}\right]^2 + 1} \right.$$

$$\tan(\beta_2 l_2) = \frac{1+a\tan\beta_1 l_1}{b\tan\beta_1 l_1 - c}, \quad \frac{m^2 Z_{01}}{Z_{02}} = \frac{1+a\tan\beta_1 l_1}{b+c\tan\beta_1 l_1}$$

$$a = \frac{-jZ_{11}}{Z_{01}}, \quad b = \frac{Z_{11}Z_{22}-Z_{12}^2}{Z_{01}Z_{02}}, \quad c = \frac{-jZ_{22}}{Z_{02}}$$

Scattering and Transmission coefs.

Expressed in terms of reflected / incident waves ~ More useful than total voltages / currents.



Normalize : $a_n = \dfrac{V_{n+}}{\sqrt{Z_{on}}}$, $b_n = \dfrac{V_{n-}}{\sqrt{Z_{on}}}$

At reference plane n:

$$V_n = V_{n+} + V_{n-} = \sqrt{Z_{on}}\,(a_n + b_n)$$

$$I_n = \frac{1}{Z_{on}}(V_{n+} - V_{n-}) = \frac{1}{\sqrt{Z_{on}}}(a_n - b_n)$$

Average power flowing into terminal n:

$$(W_n)_{av} = \frac{1}{2}\mathrm{Re}(V_n I_n^*) = \frac{1}{2}\mathrm{Re}\left[(a_n a_n^* - b_n b_n^*) + (b_n a_n^* - b_n^* a_n)\right]$$

$$\Downarrow$$

$$2(W_n)_{av} = \underbrace{a_n a_n^*}_{\text{Incident}} - \underbrace{b_n b_n^*}_{\text{Reflected}}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\Updownarrow$$

$$\underbrace{[b]}_{\text{reflected}} = \underbrace{[S]}_{\text{scattering matrix}} \underbrace{[a]}_{\text{incident}}$$

$b_1 = S_{11}\, a_1, \quad b_2 = S_{21}\, a_1$

(Source applied to port 1, Output guide matched $\sim a_2 = 0$)

Input : $2\langle W_1\rangle_{av} = (1 - S_{11} S_{11}^*)\, a_1 a_1^*$

Output : $2\langle W_2\rangle_{av} = - S_{21} S_{21}^*\, a_1 a_1^*$

(positive power $\to$ toward the port)

Passive network: $S_{21} S_{21}^* \leq 1 - S_{11} S_{11}^*$

$$F S_{11} = (Z_{11} - Z_{01})(Z_{22} + Z_{02}) - Z_{12} Z_{21}$$

$$F S_{12} = 2\sqrt{Z_{01} Z_{02}}\; Z_{12}$$

$$F S_{21} = 2\sqrt{Z_{02} Z_{01}}\; Z_{21}$$

$$F S_{22} = (Z_{22} - Z_{02})(Z_{11} + Z_{01}) - Z_{21} Z_{12}$$

$$(F = (Z_{11} + Z_{01})(Z_{22} + Z_{02}) - Z_{12} Z_{21})$$

Reciprocity : $S_{21} = S_{12}$

T (Transmission coefs) Matrix

↳ output waves in terms of input.

$$\begin{cases} b_2 = T_{11} a_1 + T_{12} b_1 \\ a_2 = T_{21} a_1 + T_{22} b_1 \end{cases}$$

$$T_{11} = S_{21} - \frac{S_{11} S_{22}}{S_{12}}, \quad T_{12} = \frac{S_{22}}{S_{12}}$$

$$T_{21} = -\frac{S_{11}}{S_{12}}, \quad T_{22} = \frac{1}{S_{12}}$$