LECTURE 27

Measurement of S/Z/Y parameters

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \quad (\text{o.c. port } 2) \quad , \quad Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \quad (\text{o.c. port } 1) \quad , \quad Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \quad (\text{s.c. port } 2) \quad , \quad Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} \quad , \quad Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

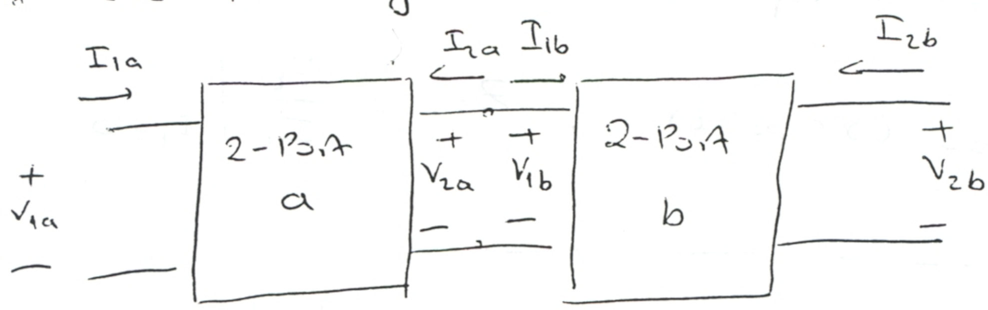
$$S_{11} = \frac{b_1}{a_1}\Big|_{a_2=0} \quad (\text{Matched port } 2) \quad , \quad S_{21} = \frac{b_2}{a_1}\Big|_{a_2=0}$$

$$S_{12} = \frac{b_1}{a_2}\Big|_{a_1=0} \quad , \quad S_{22} = \frac{b_2}{a_2}\Big|_{a_1=0}$$

Network analyzers providing
- phase and magnitude
  - ↳ vector analyzers
- magnitude only
  - ↳ scalar analyzers

Cascaded Two Ports

ABCD and T-matrix are especially useful when two ports are connected in tandem or cascade. (Output of one network becomes input of the following network)



$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}, \quad \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

and: $V_{2a} = V_{1b}$, $-I_{2a} = I_{1b}$

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \underbrace{\begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}}_{\text{Transfer Matrix of the two cascaded networks}} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

N-networks in cascade:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdots \begin{bmatrix} A_N & B_N \\ C_N & D_N \end{bmatrix}$$

Similarly for T matrix starting from the last unit, since they give output in terms of input:

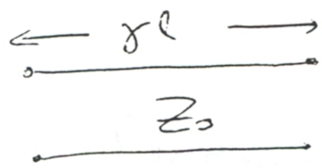
$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} (T_{11})_b & (T_{12})_b \\ (T_{21})_b & (T_{22})_b \end{bmatrix} \begin{bmatrix} (T_{11})_a & (T_{12})_a \\ (T_{21})_a & (T_{22})_a \end{bmatrix}$$

and generalizing:

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} (T_{11})_N & (T_{12})_N \\ (T_{21})_N & (T_{22})_N \end{bmatrix} \cdots \begin{bmatrix} (T_{11})_1 & (T_{12})_1 \\ (T_{21})_1 & (T_{22})_1 \end{bmatrix}$$

Examples for ABCD

(A) Transmission Line



$A = \cosh \gamma l$

$B = Z_0 \sinh \gamma l$

$C = Y_0 \sinh \gamma l$

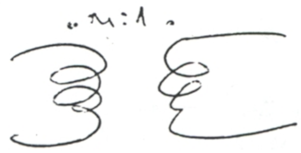
$D = \cosh \gamma l$

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\cosh^2 x - \sinh^2 x = 1$

(B) Ideal Transformer



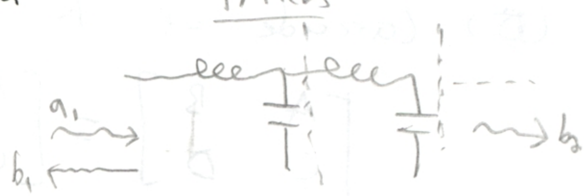
$A = n$

$B = 0$

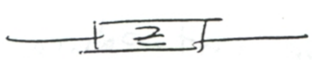
$C = 0$

$D = 1/n$

Filters



(C) Series Impedance



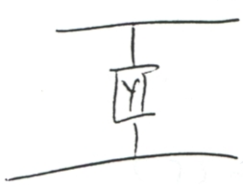
$A = 1$

$B = Z$

$C = 0$

$D = 1$

(D) Shunt ~~Impedance~~ Admittance



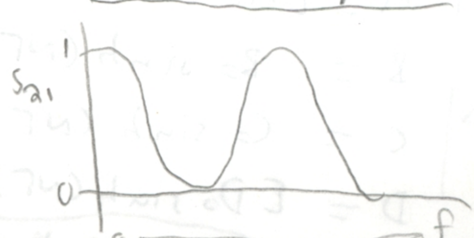
$A = 1$

$B = 0$

$C = Y$

$D = 1$

RF - filters - periodic

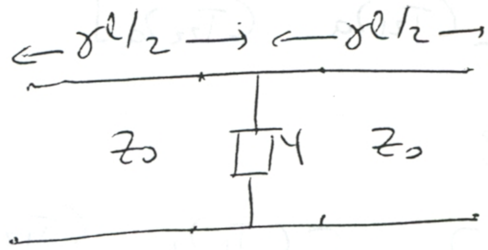


Ideally: $|S_{11}|^2 + |S_{21}|^2 = 1 \rightarrow \underset{\text{refl.}}{|S_{11}|^2} = 1 - \underset{\text{transm.}}{|S_{21}|^2}$

$|a_1|^2 = |b_1|^2 + |b_2|^2$

$1 = \dfrac{|b_1|^2}{|a_1|^2} + \dfrac{|b_2|^2}{a_1^2} = |S_{11}|^2 + |S_{21}|^2$

(E) Transm. Line with Shunt Admittance



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh\left(\frac{\gamma l}{2}\right) & Z_0 \sinh\left(\frac{\gamma l}{2}\right) \\ Y_0 \sinh\left(\frac{\gamma l}{2}\right) & \cosh\left(\frac{\gamma l}{2}\right) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} \cosh\left(\frac{\gamma l}{2}\right) & Z_0 \sinh\left(\frac{\gamma}{2}\right. \\ Y_0 \sinh\left(\frac{\gamma l}{2}\right) & \cosh\left(\frac{\gamma}{2}\right. \end{bmatrix}$$

$$\left(\begin{array}{l} \sinh(2x) = 2\sinh(x)\cosh(x) \\ \cosh(2x) = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 \\ \qquad\qquad = 1 + 2\sinh^2 x \end{array}\right.$$

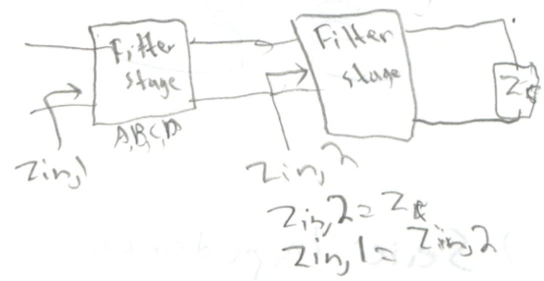
$$A = D = \cosh \gamma l + \left(\frac{Y}{2Y_0}\right) \sinh \gamma l$$

$$B = Z_0 \left[\left(\frac{Y}{2Y_0}\right) (-1 + \cosh \gamma l) + \sinh \gamma l \right]$$

$$C = Y_0 \left[\left(\frac{Y}{2Y_0}\right) (1 + \cosh \gamma l) + \sinh \gamma l \right]$$

Lect. 28

(F) Cascade of N Two-Ports



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix}^N$$

Defining: $\cosh \Gamma = (A_0 + D_0)/2$, we introduce the "effective prop. constant" of the network (charact. root of the matrix). Algebraically for a reciprocal network:

For cascade of stages

$$\begin{cases} A = [A_0 \sinh(N\Gamma) - \sinh[(N-1)\Gamma]] / \sinh(\Gamma) \\ B = B_0 \sinh(N\Gamma) / \sinh(\Gamma) \\ C = C_0 \sinh(N\Gamma) / \sinh(\Gamma) \\ D = [D_0 \sinh(N\Gamma) - \sinh((N-1)\Gamma)] / \sinh(\Gamma) \end{cases}$$

$A_0 = D_0$ symmetric; $A_0 D_0 - B_0 C_0 = 1$ reciprocal

TABLE 5.2 Conversions Between Two-Port Network Parameters

	S	Z	Y	$ABCD$
S_{11}	S_{11}	$\dfrac{(Z_{11}-Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\dfrac{(Y_0-Y_{11})(Y_0+Y_{11})+Y_{12}Y_{21}}{\Delta Y}$	$\dfrac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D}$
S_{12}	S_{12}	$\dfrac{2Z_{12}Z_0}{\Delta Z}$	$\dfrac{-2Y_{12}Y_0}{\Delta Y}$	$\dfrac{2(AD-BC)}{A+B/Z_0+CZ_0+D}$
S_{21}	S_{21}	$\dfrac{2Z_{21}Z_0}{\Delta Z}$	$\dfrac{-2Y_{21}Y_0}{\Delta Y}$	$\dfrac{2}{A+B/Z_0+CZ_0+D}$
S_{22}	S_{22}	$\dfrac{(Z_{11}+Z_0)(Z_{22}-Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\dfrac{(Y_0+Y_{11})(Y_0-Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\dfrac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D}$
Z_{11}	$Z_0\dfrac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{11}	$\dfrac{Y_{22}}{\|Y\|}$	$\dfrac{A}{C}$
Z_{12}	$Z_0\dfrac{2S_{12}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{12}	$\dfrac{-Y_{12}}{\|Y\|}$	$\dfrac{AD-BC}{C}$
Z_{21}	$Z_0\dfrac{2S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{21}	$\dfrac{-Y_{21}}{\|Y\|}$	$\dfrac{1}{C}$
Z_{22}	$Z_0\dfrac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{22}	$\dfrac{Y_{11}}{\|Y\|}$	$\dfrac{D}{C}$
Y_{11}	$Y_0\dfrac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{Z_{22}}{\|Z\|}$	Y_{11}	$\dfrac{D}{B}$
Y_{12}	$Y_0\dfrac{-2S_{12}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{-Z_{12}}{\|Z\|}$	Y_{12}	$\dfrac{BC-AD}{B}$
Y_{21}	$Y_0\dfrac{-2S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{-Z_{21}}{\|Z\|}$	Y_{21}	$\dfrac{-1}{B}$
Y_{22}	$Y_0\dfrac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{Z_{11}}{\|Z\|}$	Y_{22}	$\dfrac{A}{B}$
A	$\dfrac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$	$\dfrac{Z_{11}}{Z_{21}}$	$\dfrac{-Y_{22}}{Y_{21}}$	A
B	$Z_0\dfrac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$	$\dfrac{\|Z\|}{Z_{21}}$	$\dfrac{-1}{Y_{21}}$	B
C	$\dfrac{1}{Z_0}\dfrac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$	$\dfrac{1}{Z_{21}}$	$\dfrac{-\|Y\|}{Y_{21}}$	C
D	$\dfrac{(1-S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$	$\dfrac{Z_{22}}{Z_{21}}$	$\dfrac{-Y_{11}}{Y_{21}}$	D

$|Z| = Z_{11}Z_{22} - Z_{12}Z_{21}$; $|Y| = Y_{11}Y_{22} - Y_{12}Y_{21}$; $\Delta Y = (Y_{11}+Y_0)(Y_{22}+Y_0) - Y_{12}Y_{21}$; $\Delta Z = (Z_{11}+Z_0)(Z_{22}+Z_0) - Z_{12}Z_{21}$; $Y_0 = 1/Z_0$

LECTURE 28

Assume that each cell is symmetric : $A_0 = D_0$
and reciprocal : $A_0 D_0 - B_0 C_0 = 1$

Terminate the chain in a characteristic impedance Z_c such that each cell terminated in Z_c gives impedance Z_c at its input:

$$Z_c = \frac{V_1}{I_1} = \frac{A_0 V_2 - B_0 I_2}{C_0 V_2 - A_0 I_2} = \frac{A_0 Z_c + B_0}{C_0 Z_c + A_0}$$

$\gamma, \ell, Z_0 \quad \gamma = \Gamma, Z_0 = Z_c, \ell = 1$

$$Z_c = \left(\frac{B_0}{C_0}\right)^{1/2} \qquad (\not{+} A_0)$$

$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma\ell) & Z_0 \sinh(\gamma\ell) \\ \frac{1}{Z_0}\sinh(\gamma\ell) & \cosh(\gamma\ell) \end{bmatrix}$

replace w/ transmission line

$$A = D = \cosh N\Gamma$$

$$B = Z_c \sinh N\Gamma$$

$$C = (Z_c)^{-1} \sinh N\Gamma$$

$$\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} \qquad \cosh \Gamma = (A_0 + D_0)/2 \right)$$

(Each cell acts as a transmission line of char. impedance Z_c and overall propag. constant Γ)

Cascaded N sections have overall propag. constant $N\Gamma$.

$|\cosh \Gamma| = |(A_0 + D_0)/2|$
- $\leq 1 \Rightarrow \Gamma$ imaginary ($\Gamma = j\beta$) $\Rightarrow$ no attenuation (only time shift)
- $> 1 \Rightarrow \Gamma$ real ($\Gamma = \alpha$) and there is attenuation

passband/rejection band determined by single stage

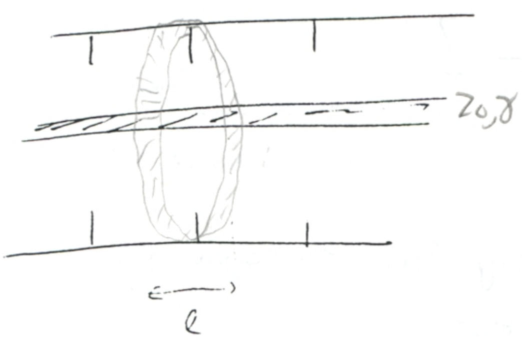
Filter!

Microwave and Optical Filters

Pass desired frequencies with small attenuation
Much higher attenuation for noise and underived signal

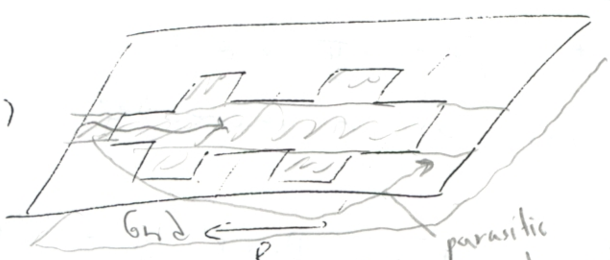
[μwave filters with periodic shunt elements]

(a)



Coaxial line with capacitive disks at intervals l, inductive - loop (small)

(b)

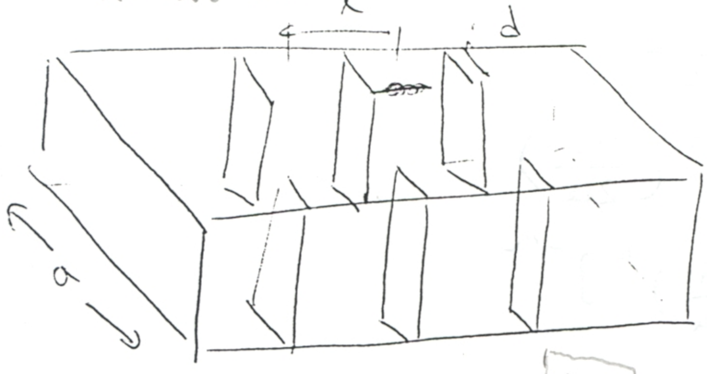


Remove mid-dielectric to reduce parasitics

Microstrip with capacitive tabs at intervals l

Almost no parasitic coupling

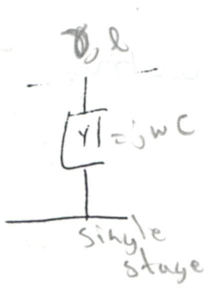
(c)



Rectangular waveguide with symmetric inductive diaphragms introduced from sides at intervals l, inductive - loop

non-symmetric inductor excites different modes - parasitics

If loading capacitors (a), (b) are C_{disk} and assume no losses → $\gamma = j\beta$ passband



$$A_0 = D_0 = \cosh\gamma l + \left(\frac{Y}{2Y_0}\right)\sinh\gamma l$$

$$\cosh\Gamma = (A_0 + D_0)/2 = \cos\beta l - \frac{\omega C_d}{2Y_0}\sin\beta l \leq 1$$

propagation

Assume L, C the inductance and capacitance of the microstrip per unit length: ③

$$\beta = \omega\sqrt{LC}, \quad Z_0 = \sqrt{L/C}$$

If $\beta l \ll 1 \xrightarrow{\text{Taylor}} \cosh\Gamma \approx 1 - \dfrac{\omega^2 l C_d \sqrt{LC}}{2\sqrt{C/L}}$

$\cos\beta l \approx 1 \quad \sin\beta l \approx \beta l$

Passband $\Leftrightarrow$ Imaginary $\Gamma \Leftrightarrow -1 \leq \cosh\Gamma \leq 1$

$$\Rightarrow [0, \omega_{c_1} = 2\sqrt{\frac{1}{C_d\, l L}}]$$

change C_d (spacing), different materials to change size of passband

cutoff angular frequency

Low Pass filter

There are other passbands at $\beta l = n\pi, n \in N$ but the passbands become narrower as n increases.

[waveguide filter with Inductive Diaphragms (c)]

Inductive ~~diagraphms~~ diaphragms

$$\frac{Y}{Y_0} = -\frac{j\lambda_g}{a}\cot^2\left(\frac{\pi d}{2a}\right)$$

$$\lambda_g = \lambda\left[1 - \left(\frac{\lambda}{2a}\right)^2\right]^{-1/2}$$

$\Downarrow$

$$\cosh\Gamma = \cos\left(\frac{2\pi l}{\lambda_g}\right) + \frac{\lambda_g}{2a}\cot^2\left(\frac{\pi d}{2a}\right)\sin\left(\frac{2\pi l}{\lambda_g}\right)$$

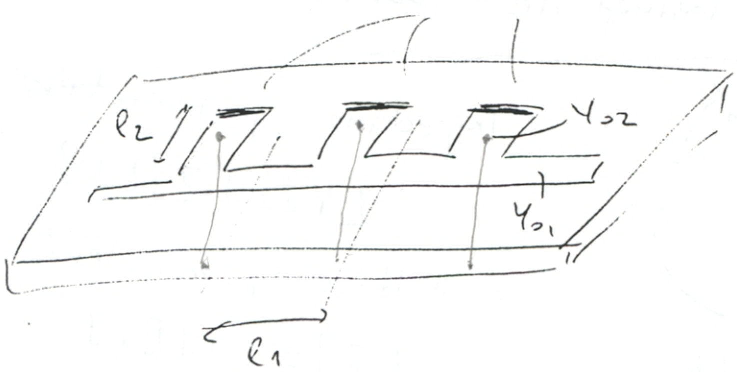
$\Rightarrow$ High-Pass filter (It remains attenuating even at higher frequencies)

Passband in the vicinity of $l = \lambda_g/2$

(Multiple passbands)

Bandpass filter in Microstrip

s.c. stubs in parallel

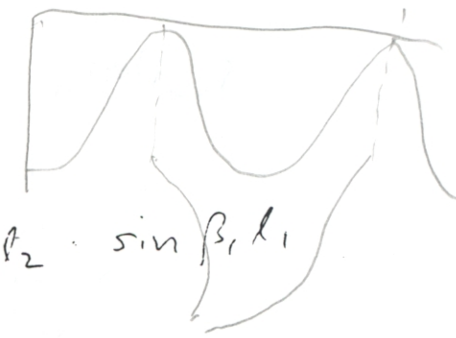


stubs can act as inductors or caps

Shunt admittances of shorted-stubs:

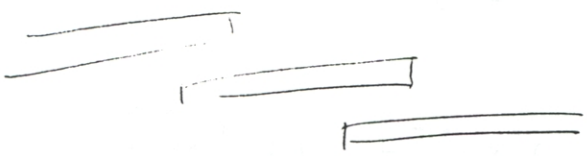
$$Y_{in} = -j Y_{02} \cot \beta_2 l_2$$

$$\cosh \Gamma = \cos \beta_1 l_1 + \frac{Y_{02}}{2 Y_{01}} \cot \beta_2 l_2 \cdot \sin \beta_1 l_1$$



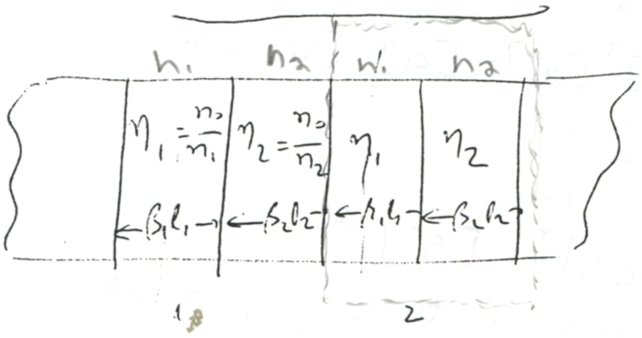
Bandpass filter for $\underset{\omega\sqrt{\mu\varepsilon}}{\beta_2} l_2$ near $\frac{(2m+1)\pi}{2}$

Filtering by Coupled μstrip lines



Optical filter.

Alternating sections of transmission line of different Z_0



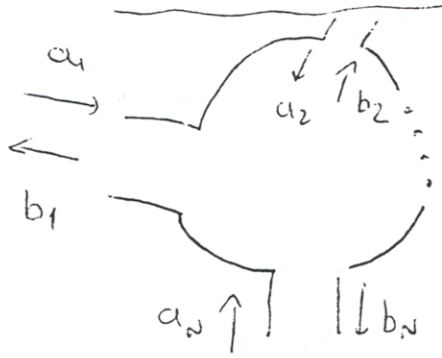
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos \frac{\beta_1 l_1}{2} & j\eta_1 \sin \frac{\beta_1 l_1}{2} \\ \frac{j}{\eta_1} \sin \frac{\beta_1 l_1}{2} & \cos \frac{\beta_1 l_1}{2} \end{bmatrix} \cdot \begin{bmatrix} \cos \beta_2 l_2 & j\eta_2 \sin \beta_2 l_2 \\ \frac{j}{\eta_2} \sin \beta_2 l_2 & \cos \beta_2 l_2 \end{bmatrix} \cdot \text{—}\,''\,\text{—}$$

$\hookrightarrow A = D = \cosh \Gamma = \cos \beta_1 l_1 \cos \beta_2 l_2 - \frac{1}{2}\left(\frac{\eta_1}{\eta_2} + \frac{\eta_2}{\eta_1}\right) \sin \beta_1 l_1 \sin \beta_2 l_2$

$\longrightarrow$ LPF

LECTURE 29

N-Port Waveguide Junctions



Impedance parameters

$$[V] = [Z][I]$$

Admittance parameters

$$[I] = [Y][V]$$

(NxN Matrices)

Scattering parameters

$$[b] = [S][a]$$

$[b]$ ↓ N waves leaving; $[a]$ ↓ N waves approaching

Reciprocal Networks

$[Z]$ $[Y]$, $[S]$ are symmetric

↳ $Z_{ij} = Z_{ji}$, $Y_{ji} = Y_{ij}$, $S_{ij} = S_{ji}$

Loss-Free Networks (lossless)

$$\text{Total Power} = \sum_{m=1}^{N} \underbrace{b_m b_m^*}_{\text{reflected}} = \sum_{m=1}^{N} \underbrace{a_m a_m^*}_{\text{incoming}} \Rightarrow [b]^T[b^*] = [a]^T[a^*]$$

$$\Rightarrow ([S][a])^T([S][a])^* = [a]^T[a^*] \Leftrightarrow$$

$$\Leftrightarrow [a]^T[S]^T[S^*][a]^* = [a]^T[a^*] = [a]^T[U][a^*]$$

$$[U]: \text{unitary matrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & 0 & 1 \end{bmatrix}$$

$$\Rightarrow [S]^T[S^*] = [U] \Rightarrow [S]^T = [S^*]^{-1}$$

(Matrices for which the transpose is the conjugate of the inverse matrix are called <u>unitary</u>)

$$\Rightarrow \begin{cases} \sum_{n=1}^{N} S_{in} S_{in}^* = 1 & \text{(I)} \\ \sum_{n=1}^{N} S_{in} S_{jn}^* = 0, \quad i \neq j & \text{(II)} \end{cases}$$

i = lines (rows)
j = columns
N = # ports

Valid for reciprocal or nonreciprocal devices!!

$\downarrow$

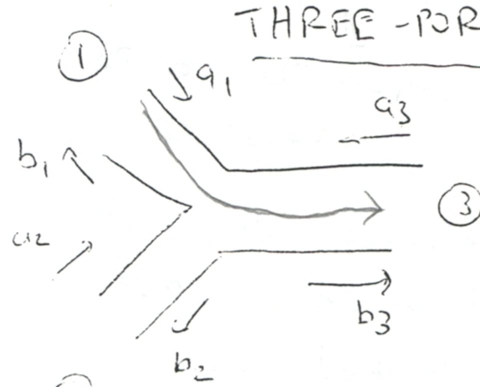
Special constraints of loss-free junctions

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Equivalently:

$[Z], [Y]$ matrices are imaginary!!

THREE-PORT Y-JUNCTION



Useful as a power divider or power combiner

(Reciprocal)

$S_{ij} = S_{ji}$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \quad \text{6 unknowns}$$

Spec. $S_{12} = 0$

Assume sources at 1, 2 with the output combined power at 3

we wish $S_{12} = 0$ (no direct interaction between two sources)

$S_{11} S_{11}^* + S_{12} S_{12}^* + S_{13} S_{13}^* = 1 \rightarrow |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$

Using (II) $\xrightarrow[j=2]{i=1}$ $S_{11} S_{21}^* + S_{12} S_{22}^* + S_{13} S_{23}^* = 0$

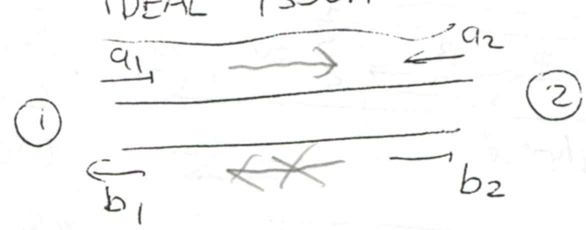
$\xrightarrow{S_{12}=0} S_{13} S_{23}^* = 0$

If $S_{12} = 0 \Rightarrow S_{13} = 0$ or $S_{23} = 0 \rightarrow$ one of the two desired couplings is eliminated. leakage from $1 \rightarrow 2$

THERE IS ALWAYS INTERACTION BETWEEN TWO SOURCES

(Use as a power divider $1 \rightarrow 2, 3$)

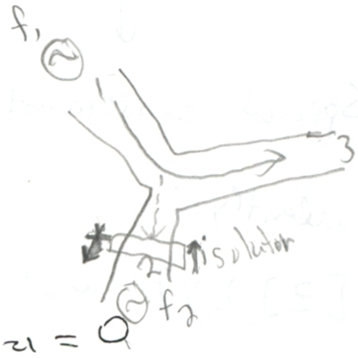
IDEAL ISOLATOR



Should be:

(·) loss-free

(·) one-way transm. line $\begin{cases} S_{12}=0 \\ S_{21}\neq 0 \end{cases}$ not reciprocal



I $\xrightarrow{i=1}$ $S_{11}S_{11}^* + S_{12}S_{12}^* = 1$ → $|S_{11}|=1$

II $\xrightarrow[j=2]{i=1}$ $S_{11}S_{21}^* + S_{12}S_{22}^* = 0$

II If $S_{12}=0$ → $S_{11}=0$ or $S_{21}=0$

I ~~[illegible]~~ $\Rightarrow S_{11}\neq 0$ → $S_{21}=0$

⇓

IMPOSSIBLE TO BE Lossless

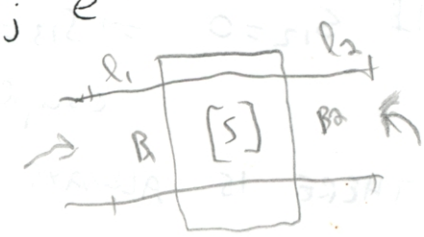
Practically: isolators use nonreciprocal elements (e.g. ferrites) but they must have dissipative elements to absorbs the reflected wave

SHIFT OF REFERENCE PLANES

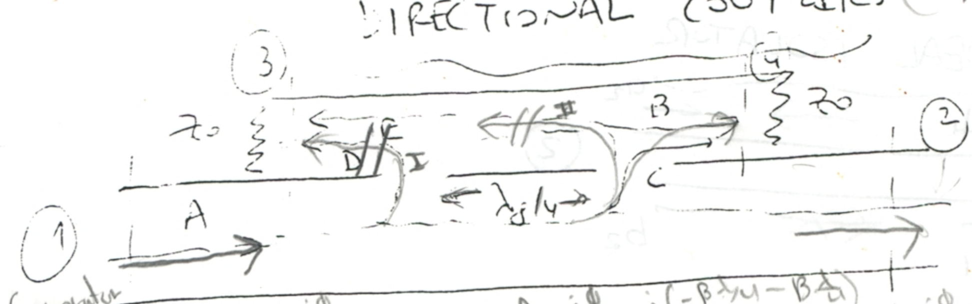
Shifting away from network by distance l_n per port

↪ additional phase delay in S-parameters

$$S'_{ij} = S_{ij}\underbrace{e^{-j\beta_i l_i}}_{\text{additional delay for } b_i}\underbrace{e^{-j\beta_j l_j}}_{\text{additional delay for } a_j} = S_{ij}\, e^{-j(\beta_i l_i+\beta_j l_j)}$$



DIRECTIONAL COUPLERS (4 ports)



Incoming at 1:
Cancel at 3
(Phase difference $\lambda_g/4 \times 2$
Couple at 4 [for equal holes]
(Same length)
(frequency sensitive)

$I = Ae^{j\phi}$ $\quad II = Ae^{j\phi}e^{j(-\beta\lambda/4-\beta\frac{\lambda}{4})} = Ae^{j\phi}e^{-j\frac{2\pi}{\lambda}\frac{\lambda}{4}\cdot 2} = Ae^{j\phi}e^{-j\pi} = -Ae^{j\phi}$

$I + II = 0$

Similarly 2 couples to 1, 3

$$[S] = \begin{bmatrix} 0 & S_{12} & \overset{S_{13}}{0} & S_{14} \\ S_{12} & 0 & S_{23} & 0\,S_{24} \\ \underset{S_{13}}{0} & S_{23} & 0 & S_{34} \\ S_{14} & \underset{S_{24}}{0} & S_{34} & 0 \end{bmatrix}$$

4 unknowns

no reflection b/c. terminate w/ Z_0

Assume negligible loss $\Rightarrow$ S will be unitary

$i=1 \Rightarrow \underbrace{S_{12}S_{12}^* + S_{14}S_{14}^*}_{|S_{12}|^2 + |S_{14}|^2} = 1$ (III)

$i=2 \Rightarrow S_{12}S_{12}^* + S_{23}S_{23}^* = 1$ (IV)

$i=3 \Rightarrow S_{23}S_{23}^* + S_{34}S_{34}^* = 1$ (V)

$i=4 \Rightarrow \underbrace{S_{14}S_{14}^* + S_{34}S_{34}^*}_{|S_{14}|^2 + |S_{34}|^2} = 1$ (VI)

III, IV $\Rightarrow |S_{14}| = |S_{23}|$

III, VI $\Rightarrow |S_{12}| = |S_{34}|$

Choose reference 2 with respect to 1 : S_{12} : positive real

4 ← 3 : S_{34} : positive real

$$S_{12} = S_{34} \triangleq a$$

Taking the zero equations (II)

$i=1, j=3 \Rightarrow S_{12}S_{23}^* + S_{14}S_{34}^* = 0$

$i=2, j=4 \Rightarrow S_{12}S_{14}^* + S_{23}S_{34}^* = 0$

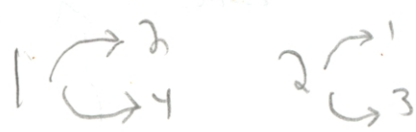
$\Rightarrow S_{14} = -S_{23}^*$

$\xrightarrow[S_{14} = \text{real}]{\text{Choose } 4 \text{ to } 1}$ $S_{23} = -S_{14} \triangleq b$

$$[S] = \begin{bmatrix} 0 & a & 0 & -b \\ a & 0 & b & 0 \\ 0 & b & 0 & a \\ -b & 0 & a & 0 \end{bmatrix}$$

a : transmission ~~coupling~~ factor

b : coupling factor to the auxiliary guide



(I) $\Rightarrow$ $\boxed{a^2 + b^2 = 1}$

For real couplers: $S_{13}, S_{24} \neq 0$

Front-to-back ratio (Directivity) $= \frac{P_{1\to 4}}{P_{1\to 3}}$ Ideally $= \infty$

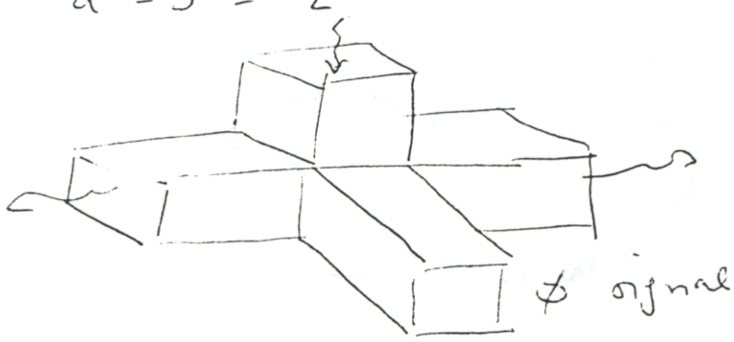
$= \frac{\text{Coupling to the desired terminal of auxiliary guide}}{\text{— " — undesired — " —}}$

(.) A 4-port with two pairs of noncoupling elements is completely matched.

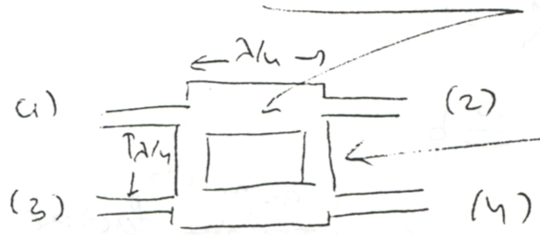
$S_{13} = 0 = S_{24} \Rightarrow S_{11} = S_{22} = 0$

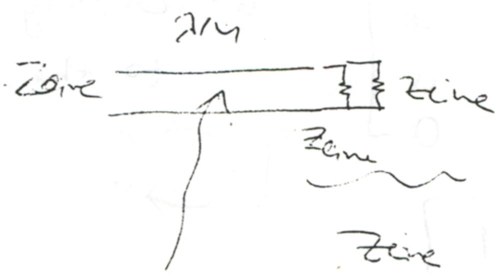
Magic-T

$a^2 = b^2 = 1/2$



"Rat race"





$\lambda/4$-Transformer

$\sqrt{Z_{line} Z_{line}/2} = Z_{line}/\sqrt{2}$