**Problem 1:**

\[ Y(x,t) = 1.5 \cos \left( 2\pi \cdot 10^8 t - \frac{2\pi}{3} x + \frac{\pi}{3} \right) e^{-0.0005x} \text{ (Volts)} \]

(a) The Amplitude is 1.5 Volts at spatial position \( x = 0 \text{m} \). It has an attenuation of \( e^{-0.0005x} \).

The Period, \( T = \frac{2\pi}{2\pi \cdot 10^8} = 10^{-6} \text{s} \)

The frequency is \( 1/T = 10^8 \text{ Hz} = 100 \text{ MHz} \)

The wavelength, \( \lambda = c/f = 3 \text{ m} \)

The phase, \( \phi = \pi/3 \text{ rad} \)

The attenuation factor, \( \alpha = 0.005 \text{ Np/m} \) (assuming the negative sign is intrinsic to \(+x\) prop).

(b) \( \beta = \frac{2\pi}{\lambda} = \frac{2\pi}{3} \text{ (rad/m)} \)

\( \omega = 2\pi f = 2\pi \cdot 10^8 \text{ (rad/s)} \)

(c) The wave propagates in the \(+x\) direction due to negative sign in front of \( \beta \). This occurs due to a phase front needing to remain constant – increasing time requires a positive increase in \( x \) to maintain neutrality with time for the phase front.

(d) The amplitude at \( t = 10^{-5} \text{s} \) and \( x = 3000 \text{m} \) is given by \( 1.5 \cos(2000\pi - 2000\pi + \pi/3)e^{-1.5} \)

This value is .167 Volts which satisfies the .1V requirement for reception.

**Problem 2:**

(a) \( L = 0.1 \text{m}, f = 1 \text{ GHz} \Rightarrow \lambda = 0.3 \text{m}, \text{ so } L/\lambda = 1/3 >> 0.01 \Rightarrow \text{do not ignore tline effects} \)

(b) \( L = 1 \text{m}, f = 1.8 \text{ GHz} \Rightarrow \lambda = 0.1667 \text{m}, \text{ so } L/\lambda = 6 >> 0.01 \Rightarrow \text{do not ignore tline effects} \)

(c) \( L = 0.01 \text{m}, f = 0.9 \text{ GHz} \Rightarrow \lambda = 0.333 \text{m}, \text{ so } L/\lambda = 0.03 > 0.01 \Rightarrow \text{do not ignore tline effects (close)} \)

(d) \( L = 0.05 \text{m}, f = 60 \text{ GHz} \Rightarrow \lambda = 0.005 \text{m}, \text{ so } L/\lambda = 10 >> 0.01 \Rightarrow \text{do not ignore tline effects} \)
**Problem 3:**

(a) \( \Gamma = \frac{30-j60-50}{30-j60+50} = .2 - .6j \)

(b) \( \text{SWR} = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+.6325}{1-.6325} = 4.4415 \)

(c) \( Z_{in}(.35\lambda) = \frac{(30-j60)+j50\tan(7\pi)}{50+j(30-j60)\tan(7\pi)}(50) = 78.4644 + j98.2431 \)

(d) Under a perfect match, 10W can be delivered. This can be interpreted as the input power to the system is 10W. For the match at hand, \( |\Gamma| = .6325 \). Thus, the power received is given by \( (1-|\Gamma|^2)P_{in} = 6W \)

**Problem 4:**

(a) \( Z_o = 50\Omega, L = \lambda/4, Z_L = 60\Omega \). Find the input impedance. For a quarter-wave length section, the input impedance is given by \( Z_{in} = Z_o^2/Z_L = 2500/60 = 41.67\Omega \)

(b) For an open circuit, \( \Gamma = 1 \). For an open circuit \( Z_{in} = -jZ_o\cot(\beta l) \). So for \( L = \lambda/8 \), \( Z_{in} = -j50\Omega \)
   This circuit behaves as a capacitor due to the negative complex impedance.

(c) For a short circuit, \( \Gamma = -1 \). For a short circuit, \( Z_{in} = jZ_o\tan(\beta l) \). For for \( L = \lambda/6 \), \( Z_{in} = j86.6\Omega \)
   This circuit behaves as an inductor due to the positive complex impedance.

**Problem 5:**

(a) Because the load is a short circuit, the reflection coefficient is -1.

(b) Because the input impedance is dependent on a tangent function, it is periodic with \( \pi \). Thus, every \( .5\lambda \) returns to the load impedance. Thus, \( 2.3\lambda \) is equivalent to \( .3\lambda \). So, \( Z_{in} = j50\tan(.6\pi) = -j153.88\Omega \).

(c) The input admittance is given by \( 1/Z_{in} = .0065j \) S (Siemens or \( \Omega^{-1} \)).
Problem 6:

\( Z_o = 50 \, \Omega \)
\( Z_l = 75 - j20 \, \Omega \)
\( f_o = 6 \, \text{GHz} \)

(a) Because the real component of the load is larger than \( Z_o \), the correct circuit topology to be used is:

Using the design equations,

\[
B = \frac{X_L \pm \sqrt{R_L/Z_o \left( R_L^2 + X_L^2 - Z_o R_L \right)}}{R_L^2 + X_L^2} = .006373, -.013015
\]
\[
X = \frac{1}{B} + \frac{X_L Z_o}{R_L} = \frac{Z_o}{B R_L} = 38.9444, -38.9444
\]

For (.006373, 38.9444)

\[
L = \frac{X}{\omega} = \frac{38.9444}{2\pi \cdot 6 \cdot 10^9} = 1.033 \, \text{nH}
\]
\[
C = \frac{B}{\omega} = \frac{.006373}{2\pi \cdot 6 \cdot 10^9} = .16913 \, \text{pF}
\]

For (-.013015, -38.9444)

\[
L = \frac{1}{\omega B} = 2.0381 \, \text{nH}
\]
\[
C = \frac{1}{\omega X} = .68112 \, \text{nF}
\]

\( Z_{in} = j\omega L + j\omega C||Z_l \)

This results in the following gamma plot:

Using the cursor, \( \Delta f = 24.45\% \, (0.05 \, \Gamma_m) \)
For the second set (2.0381 nH, .68112 pF)

\[ Z_{in} = -\frac{j}{\omega C} + \left(\frac{-j}{\omega L}\right) || Z_L \]

This results in the following gamma plot:

![Gamma Plot](image)

Using the cursors, \( \Delta f = 18.02\% \)

(b) \( Z_L = 25-j20 \, \Omega \). This results in the second circuit topology for matching:

![Circuit Diagram](image)

Using the design equations:

\[
X = \pm \sqrt{R_L(Z_L - R_L)} - X_L = 45, -5 \\
B = \pm \sqrt{\frac{(Z_o - R_L)/R_L}{Z_o}} = \pm .02
\]

This results in the first LC pair as \((B, X) \Rightarrow L = \frac{X}{\omega} = 1.1937 \, nH \\
C = \frac{B}{\omega} = .53052 \, pF\]

This produces a \( Z_{in} \) of

\[ Z_{in} = j\omega C || (j\omega L + Z_L) \]
This additionally yields the following gamma plot

![Gamma Plot](image)

This results in a $\Delta f = 9.7\%$

The second pair of LC (for the negative values)

\[
L = \frac{1}{\omega B} = 1.3263 \, nH
\]
\[
C = \frac{1}{\omega X} = 5.3052 \, pF
\]

This produces a $Z_{in}$ of:

\[
Z_{in} = \left(\frac{-j}{\omega L}\right) || \left(\frac{-j}{\omega C} + Z_L\right)
\]

This yields a gamma plot of:

![Gamma Plot](image)

This produces $\Delta f = 19.85\%$
Problem 7:

\[ f = 100 \text{ MHz}, \quad Z_o = 300 \Omega, \quad Z_L = 73 \Omega \]

(a) Quarter-Wavelength Transformer (electrical length \( \lambda/4 \))

\[ Z_{\lambda/4} = \sqrt{Z_o Z_L} = 147.99 \Omega \]

(b) Determine the physical length:

\[ \lambda = \frac{C_o}{\sqrt{\varepsilon_r f}} = \frac{3 \cdot 10^8}{1.6 \cdot 10^8} = 1.875 \text{ m} \Rightarrow \frac{\lambda}{4} = 0.4688 \text{ m} \]

(c) Determine the bandwidth given \( \Gamma_m = 0.05 \)

\[ \frac{\Delta f}{f_o} = 2 - \frac{4}{\pi} \cos^{-1} \left[ \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_o Z_L}}{|Z_o - Z_L|} \right] = 2 - \frac{4}{\pi} \cos^{-1} \left[ \frac{0.05}{\sqrt{1 - 0.05^2}} \frac{2\sqrt{300 \cdot 73}}{|300 - 73|} \right] = 8.32\% \]

(d) Use a 3 Stage Binomial Transformer to Design a more bandwidth efficient match:

The recursive formula for generating the section impedances is:

\[
\ln \left( \frac{Z_{n+1}}{Z_n} \right) = 2^{-N} C_n^N \ln \left( \frac{Z_L}{Z_o} \right)
\]

\[ Z_o = 300 \Omega \]
\[ Z_1 = 251.42 \Omega \]
\[ Z_2 = 148 \Omega \]
\[ Z_3 = 87.11 \Omega \]
\[ Z_L = 73 \Omega \]

(e) The bandwidth for this system is given by (N=3):

\[ \frac{\Delta f}{f_o} = 2 - \frac{4}{\pi} \cos^{-1} \left( \frac{\frac{\Gamma_m}{2}}{\frac{Z_L - Z_o}{2^{N}(Z_L + Z_o)}} \right) = 57.3\% \]