Intracell Modeling of Metal/Dielectric Interfaces for EBG/MEMS RF Structures Using the Multiresolution Time-Domain Method

Nathan A. Bushyager and Manos Tentzeris
Georgia Institute of Technology
Atlanta, GA 30332
nbushyager@ece.gatech.edu

Abstract: A method that allows the intracell modeling of a PEC/dielectric interface using MRTD is presented in this paper. This approach involves zeroing scaling and wavelet functions that intersect the metal, and results in the decoupling of the fields on either side of the metal. Applications of this technique to EBG patterned ground planes and RF-MEMS are discussed.
Keywords: MRTD, FDTD, MEMS, EBG

1. Introduction

Modern microwave structures are often tedious to simulate because they consist of large, distributed, high-frequency elements that need to be simulated using a full-wave method. This is difficult using most conventional techniques because the high aspect ratio of the components requires the use of small elements and leads to the use of extremely large grids. In order to simulate modern circuits, which may consist of MEMS and EBG structures, a method that allows the use of an adaptive grid is preferable. In this paper a method is presented that enables the multiresolution time-domain (MRTD) [1] technique to accurately represent metals intersecting the grid. Using this method, high resolution MRTD cells can accurately model areas with complex metal arrangement, while low resolution MRTD cells can be used elsewhere. This can be accomplished using the MRTD technique by zeroing out wavelet and scaling coefficients that intersect the metal. The higher resolution elements which exist on either side of the metal are then decoupled, modeling the boundary condition of a PEC that exists within the cell. The resolution required in these cells can then be determined through the use of thresholding, where the relative and absolute magnitude of the wavelet coefficients can be used to determine which resolution is required to accurately simulate the structure. One example of these type of structures is a patterned ground plane for a multilayer topology, that acts as a shield between layers for selected frequencies while being mechanically compatible with the manufacturing process. A second example involves the hybrid electromagnetic and mechanical simulation of a MEMS switch with a moving membrane employing time varying intracell modeling.

2. Background

The multiresolution time-domain technique (MRTD) is so named because it employs multiresolution principles to discretize Maxwell's equations in a wavelet expansion. Wavelet expansions provide a set of functions with adaptive resolution. Higher resolution functions can be added and subtracted during simulation. Due to this, the MRTD technique has a built in time- and space- adaptive gridding capability. When applied to Maxwell's curl equations, the orthogonality of the wavelets provides an efficient discretization which leads to an explicit time marching scheme much like the finite-difference time-domain technique (FDTD) [2]. The expansions of E and H are presented in (1) and (2) for a 1D scheme consisting of E_z and H_y fields propagating in the x direction. In these equations the location of the coefficients in time and space for the electric field are denoted by n and m, respectively. The time and space locations of H are denoted by n' and m'. The relationship between the primed and unprimed parameters of these coefficients depends on the choice of basis functions used. The update equations for the MRTD scheme can be determined by inserting (1) and (2) into (3) and (4) and applying the method of moments.

$$E_z(x) = \sum_{n,m} h_{n(t)} \left[{}_nE_m^{z,\phi} \varphi_m(x) + \sum_r \sum_p {}_nE_{m,r,p}^{z,\psi} \psi_{m,p}^r \right] \tag{1}$$

$$H_y(x) = \sum_{n',m'} h_{n'}(t) \left[H_{m'}^{y,\phi} \varphi_{m'}(x) + \sum_r \sum_p {}_{n'}H_{m,r,p}^{y,\psi} \psi_{m',p}^r \right] \tag{2}$$

$$\frac{\partial}{\partial t} E_z(x,t) = \frac{1}{\varepsilon} \frac{\partial}{\partial x} H_y(x,t) \tag{3}$$

$$\frac{\partial}{\partial t} H_y(x,t) = \frac{1}{\mu} \frac{\partial}{\partial x} E_z(x,t) \tag{4}$$

Fig. 1 Haar scaling function

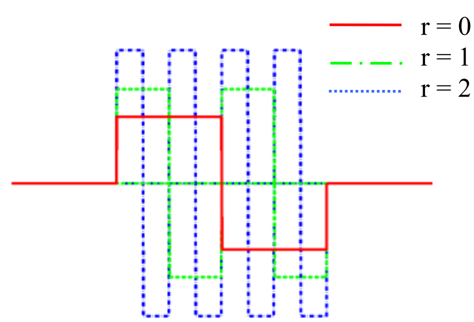

Fig.2 Haar wavelets, r=0,1,2

There are many families of wavelets that can be used in the above equations. The time discretization is usually performed using pulse functions, like that presented in Fig. 1, called Haar scaling functions. This makes the time discretization relatively simple and avoids non-causality which can arrive through the use of entire domain functions. Wavelets are usually chosen for their ability to produce very sparse matrices when used to discretize equations. For this investigation, the Haar scaling and wavelet functions, presented in Figs. 1 and 2 will be used [3]. While not as efficient as other wavelet expansions, the Haar wavelets have the advantages of being both easy to work with and having a finite domain. This nature of the Haar wavelets makes it possible to easily deal with hard boundaries such as PEC (and PMC) boundary conditions.

Another advantage of using Haar wavelets is that the derivative of the Haar wavelets and scaling function is a series of delta functions. These delta functions make the calculation of the inner products when applying the method of moments straightforward, and no quadrature is required during simulation. When (1) and (2) are inserted into (3) and (4) and the method of moments (Galerkin) is applied, independent update equations are obtained for each scaling and wavelet function. Each E (H) coefficient is only dependent on its previous value, and the values of the surrounding E (H) coefficients. For example, the update equation for the H scaling function is

$$ {}_{n'+1}H^{y,\phi}_{m'} = {}_{n'}H^{y,\phi}_{m'} + \frac{\Delta t}{\mu \Delta x}\left({}_{n}E^{z,\phi}_{m+1} - {}_{n}E^{z,\phi}_{m} + \sum_{r=0}^{r\max} 2^{r/2}\left({}_{n}E^{z,\psi}_{m+1,r,0} - {}_{n}E^{z,\psi}_{m,r,0} \right) \right) \qquad (5). $$

It has been shown [4] that the offset between the E and H cells should be $1/2^{\mathrm{rmax}+2}$ where rmax represents the highest resolution used in the simulation. Using the notation from (1) and (2) $m' = m + 1/2^{\mathrm{rmax}+2}$.

One of the important features of the MRTD method is that the cells used in the discretization are larger than those used in a similar FDTD grid. In many FDTD simulations, however, the grid size is determined not by the stability requirements, but rather by the feature size of the structure being modeled. For example, and EBG structure may consist of a periodic arrangement of multiple dielectrics. If each cell contains only one dielectric, then MRTD will offer little to no advantage over FDTD. Therefore, it is necessary to use a scheme that allows the dielectric constant to vary over the length of a cell [5]. In the derivation of the method, the D field is not replaced by the product of a fixed ε and E, instead D is solved for directly. A multiplication of the D coefficients with a transformation matrix based on the ε spatial distribution is then used to determine the E fields. The coefficients of this matrix are calculated before simulation, and the method therefore has minimal impact on the speed of the scheme.

3. Subcell Modeling Techniques

In addition to the time and space adaptive grid, MRTD allows the use of cells closer in size to the Nyquist limit [1]. While all of these cells will contain wavelet functions, cells far from discontinuities with low field variation will not contain many wavelets with significant values. An adaptive scheme that compares the values of these wavelets to their respective scaling function can be used to determine wavelets that can be safely neglected in the update equations [6]. In order to use these large cells, however, a method that enables metals to intersect a cell must be used. Using such methods, an MRTD grid can be applied to an arbitrary geometry.

Subcell modeling methods have been applied in the past to FDTD [7],[8]. One application is for a MEMS structure, where the membrane height may be on the order of a few microns. If this structure is inserted into a larger circuit, on the order of a few thousand microns, the grid that conforms to this small feature can easily have millions of cells and take days to execute. The subcell modeling techniques allow the grid resolution to be increased around the MEMS circuit, while the lower resolution is used elsewhere.

While these techniques allow some degree of adaptability to the grid, they have several limitations. First, they allow the resolution to be varied in space, but not time. Second, they introduce special update equations in the area of the discontinuity. These modified update equations utilize interpolation to add grid points in areas where they do not naturally exist. These interpolations can add to grid dispersion. In contrast, variable resolution is a built-in feature of MRTD. In the next section, a method to insert metals inside these cells will be presented.

4. MRTD Intracell Metal Modeling

When inserting a PEC into an FDTD or MRTD grid, the boundary condition that must be enforced is that electric fields tangential to the metal must be set to zero. This is a natural condition in FDTD, as metals can be placed along cells that coincide with the electric field locations in the Yee cell. This condition can be exactly duplicated in Haar MRTD by placing metals along the electric field locations in the modified Yee cell that represents the MRTD grid. If a metal only covers a portion of the cell, only the scaling and wavelet functions that intersect the metal need to be zeroed. By increasing the resolution, a metal intersecting any part of the grid can be represented. An example of this is presented in Fig. 3. In this case the metal splits a cell in two. The scaling function is intersected by the metal. It must be set to zero. The 0^{th} resolution wavelet, however, is also split by the metal. Instead of zeroing it, it can be duplicated, one wavelet for each side of the metal employing image theory. For update equations for this wavelet, contributions from the other side of the metal are assumed to be zeroed. In this way the wavelet acts as a scaling function for half of the cell.

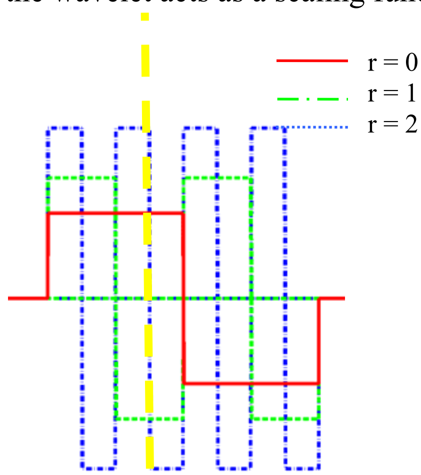

Fig. 3 Metal intersecting the center of a Haar MRTD cell

This method can be especially useful for the modeling of RF-MEMS devices, structures consisting of moving membranes which change their position with time. The time-adaptive MRTD grid is computationally efficient in this case as the wavelet functions that are set to zero can be made to coincide with the location of the metal at any given time step.

5. Conclusion

In this paper a method was presented that enables the modeling of a metal that cuts through an MRTD cell. This technique is very important for the modeling of finely detailed structures using the MRTD method since it allows for the use of a fixed physical cell size (scaling function) and the adaptive implementation of a denser effective cell through the use of different resolutions of wavelet functions. This gridding allows the modeling of RF-MEMS devices, because the variable resolution accurately represents the motion of the membrane in the time domain. It also allows for the simulation of patterned ground planes that consist of intracell metal and dielectric interfaces.

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