# Composite Cell MRTD Method for the Efficient Simulation of Complex Microwave Structures

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***Abstract* — This paper presents a subcell modeling method for use with the Haar-MRTD technique. This technique enables the application of the PEC boundary condition at individual equivalent grid points. Through the use of field reconstruction/decomposition, a method is presented that allows the application of pointwise effects using the MRTD technique. This technique makes it possible to use the MRTD time- and space-adaptive grid in the simulation complex structures consisting of PECs and other local effects.**

## I. INTRODUCTION

The multiresolution time-domain (MRTD) [1] technique makes possible the application of wavelet decomposition principles to the space discretization of Maxwell's equations. The practical outcome of this application is a time- and space- adaptive grid. This grid is very useful because the resolution can be customized to match a given structure using a minimum number of grid points. Through careful application of thresholding [2], this effect can be enhanced by allowing the resolution to be changed based on the requirements of representing the waveform as a function of time. The use of wavelets, however, is not without a price and the application of localized effects becomes difficult.

The quantities that are found directly in MRTD are the values of the wavelet/scaling coefficients; the field values must be reconstructed by summing all coefficients that overlap at any given point. Because all wavelet coefficients cover multiple grid locations, the application of effects at individual points in the grid is challenging. Changing the values of wavelet coefficients to alter the field values at one point effects the values at many other points. Careless coefficient modification can lead to non-physical field values and unstable algorithms.

The Haar wavelet family is in many ways one of the simplest, however, it has many properties that make its application to practical structures favorable [3]. Most importantly, it is finite-domain and when reconstructed leads to finite areas of constant field value (equivalent grid points [4]). Using this property, it is possible to apply pointwise effects in the MRTD grid when an arbitrary level of Haar wavelets is used.

This paper presents a technique that can be used with Haar-MRTD to apply PEC effects at individual equivalent grid points. This technique uses wavelet reconstruction/decomposition to apply pointwise effects in the MRTD grid. This technique makes possible the use of the MRTD time- and space-adaptive grid for complex structures. In addition, each MRTD cell can contain complex PEC structures. Using this method, large, sparse cells can be used in homogeneous areas surrounding high resolution structures while high resolution grids can be used to represent fine features.

## II. BACKGROUND

The technique presented in this paper is as much a way of furthering understanding of the MRTD method as an extension to model PEC elements. To clearly present the method, a brief derivation of 2D Haar-MRTD is presented, as well as a partial listing of the properties of Haar wavelet expansions.

### *A. Haar Basis Functions*

Haar basis functions are based on pulses in space. The Haar scaling function, $\phi$, as well as the Haar mother wavelet, $\psi_0$, are presented in Fig. 1. The scaling function is simply a pulse function over a given domain. The wavelet function is based on the scaling function, and consists of two pulses, each of half the domain of the scaling function and of the opposite magnitude. The inner product of either function with itself is 1, while the inner product of the two functions is 0 [3].

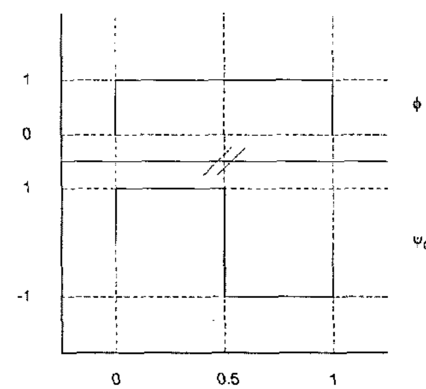


Fig. 1 Haar scaling function, $\phi$, and mother wavelet $\psi_0$

The Haar wavelets of higher resolution levels are based on the mother wavelet. For each level of resolution the number of wavelets is doubled while the domain of each is halved. The magnitude of each function is modified so that the inner product of each wavelet function with itself is one. The inner product of any wavelet coefficient with any other wavelet coefficient, at any resolution level, or with the scaling function, is 0 [3]. Fig. 2 presents the wavelet coefficients for wavelet resolution levels 1 and 2.

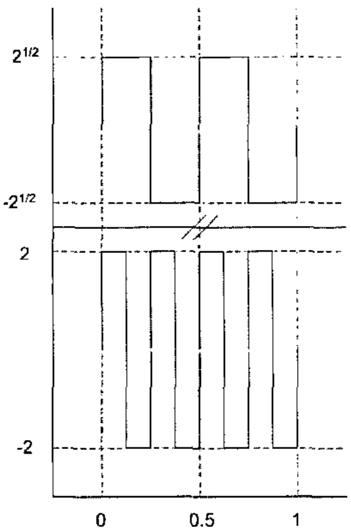


Fig. 2 Haar wavelets at resolutions 1 and 2

The reconstruction of the wavelets yields some interesting properties. When the coefficients of the expansion are summed to determine field values, the function appears as a pulse train. The pulses have the domain of half of the highest resolution wavelet. Furthermore, these pulses overlap the constant valued sections of the highest resolution wavelets. A linear combination of the wavelet/scaling functions has as many degrees of freedom as the number of coefficients used. There are $2^{r_{max}+1}$ functions used per level, and any finite real value can be represented at the center of each half of the $r_{max}$ level wavelets.

The effect of the variable grid when it is used to represent electromagnetic fields can be easily seen. If the field value can be approximated as constant across the half-domain of the highest resolution wavelet, there is no need for increasing resolution. If the field has more rapid variation, each increase in resolution doubles the effective resolution of the cell. High resolution cells can be used to represent rapid field variation (such as impressed currents and discontinuity effects) while low resolution cells can be used elsewhere.

*B. Haar-MRTD Derivation*

The equations,

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\frac{\partial H_z}{\partial y} \quad (1)$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\varepsilon}\frac{\partial H_z}{\partial x} \quad (2)$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left[\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right] \quad (3)$$

represent the 2D $TE_z$ mode of Maxwell's equations for source-free, lossless, isotropic media. These functions will be used to demonstrate the expansion of Maxwell's equations in this paper as a compromise between completeness and space requirements.

The expansion of the $E_x$ field in (1) in terms of Haar scaling and wavelet functions is

$$E_x(x) = \sum_{n,i,j} h_{n(t)} \begin{bmatrix} {}_nE^{x,\phi\phi}_{i,j}\varphi_i(x)\varphi_j(y) + \\ \sum_{r=0}^{r_{max}}\sum_{p=0}^{2^r-1} {}_nE^{x,\psi\phi}_{i,j,r,p}\psi^r_{i,p}(x)\varphi_j(y) + \\ \sum_{r=0}^{r_{max}}\sum_{p=0}^{2^r-1} {}_nE^{x,\phi\psi}_{i,j,r,p}\varphi_i(x)\psi^r_{j,p}(y) + \\ \sum_{r=0}^{r_{max}}\sum_{p=0}^{2^r-1}\sum_{s=0}^{r_{max}}\sum_{q=0}^{2^s-1} {}_nE^{x,\psi\psi}_{i,j,r,p,s,q}\psi^r_{i,p}(x)\psi^s_{j,q}(y) \end{bmatrix} \quad (4)$$

In a 2D expansion wavelets and scaling functions are used in both the $x$ and $y$ directions. The terms in (4) represent the products of the basis functions in both directions. For each of these products, one coefficient results. The four groups of coefficients represent the scaling-$x$/scaling-$y$, wavelet-$x$/scaling-$y$, scaling-$x$/wavelet-$y$, and wavelet-$x$/wavelet-$y$ coefficients. There are $2^{2(r_{max}+1)}$ wavelets for a maximum resolution $r_{max}$. For a maximum resolution level $r_{max}$=0, the four coefficients in 2D (one for each product term in (4)) are presented in Fig. 3.

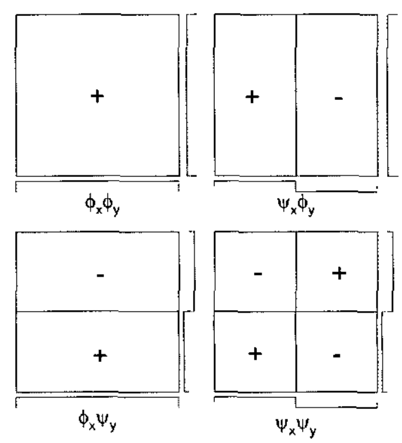


Fig. 3 2D Haar coefficients for $r_{max}$=0

When the E and H field expansions are inserted into (1)-(3) the method of moments can be applied to determine update equations for each of the wavelet/scaling coefficients [3]. It has been shown [3,4] that the offset $1/2^{r_{max}+2}$ between the E and H fields in this expansion yields the best dispersion properties and locates the equivalent grid points in the same pattern as the FDTD-Yee cell [5]. In the 2D case, like the previously presented 1D case, the equivalent grid points are at the center of the constant valued sections of the highest resolution wavelets. In Fig. 3 these are the locations of the + and – in the $\psi_x\psi_y$ function.

The update equations for this case are

multiplying the electric field vectors in (13) with $\mathbf{I_p}$ is redundant.

The new update equation with PEC locations zeroed is

$$\mathbf{R}_n\mathbf{E}^x_{i,j} = \mathbf{R}_{n-1}\mathbf{E}^x_{i,j} + \frac{\Delta t}{\varepsilon\Delta y}\left(\mathbf{I_p}\mathbf{U}'_{E_{x1}\,n-1}\mathbf{R}\mathbf{H}^z_{i,j} + \mathbf{I_p}\mathbf{U}'_{E_{x2}\,n-1}\mathbf{R}\mathbf{H}^z_{i,j-1}\right) \quad (14)$$

By multiplying (14) with $\mathbf{R}^{-1}$ and defining

$$\mathbf{U}^p = \mathbf{R}^{-1}\mathbf{I_p}\mathbf{U}'\mathbf{R} \quad (15)$$

the PEC MRTD update equation is

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This equation is the same as (5) except for the use of the $\mathbf{U}^p$ matrices. Thus, it is possible to implement subcell PEC modeling in MRTD while only changing the inner product matricies. This method adds no increase in computational overhead, and simply requires the additional memory to store the **U** matricies.

## IV. EXAMPLE

To test the method, a PEC screen in a 2D parallel plate waveguide was simulated. An expanded view of the grid surrounding the screen is presented in Fig. 5. The areas where the PEC is applied are shaded. A maximum wavelet resolution of 2 was used. In this simulation, the voltage at the output of the screen was probed. When compared to the results of an FDTD simulation of the same structure, Fig. 6, the maximum difference in magnitude is shown on the order of $10^{-12}$% of the peak field value. This is within the numerical accuracy of Matlab [7], which was used for the simulations, showing that the techniques are identical.

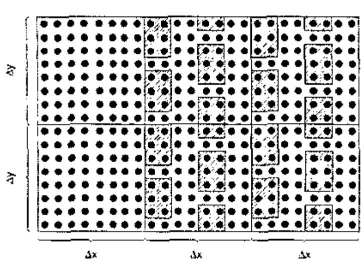


Fig. 5 Grid used to represent PEC screen

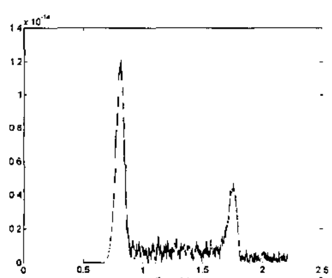


Fig. 6 Difference between FDTD and MRTD results

## V. CONCLUSION

This paper presents a technique that can be used to apply local effects on the Haar-MRTD grid. The domain of the effect is that of one equivalent grid point. By increasing the resolution arbitrarily small features can be modeled. One application of this technique is the application of subcell PECs.

This technique allows the time- and space- adaptive grid of MRTD to be used to model finely detailed structures. Areas of the grid containing small features can use increased resolution, while homogenous areas can use low resolution. It is important to note that this technique can be used to model structures with continuous dielectric variations, and thus composite cells, that is those with multiple PEC and dielectric regions per cell, can be modeled. Furthermore, the pointwise expansion of the MRTD equations can be used in the future to model other subcell components, such as lumped elements and equivalent circuits.


## ACKNOWLEDGEMENT

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[6] T. Dogaru and L. Carin, "Scattering analysis by the multiresolution time-domain method using compactly supported wavelet systems," IEEE Trans. Microwave Theory Tech, vol. 50, no. 7, pp.1752-1760 July 2002.

[7] Matlab, The MathWorks, 3 Apple Hill Drive, Natick, MA 01760-2098

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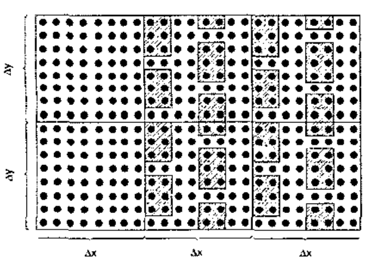

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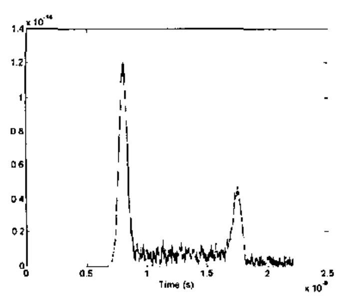


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