ECE 3065
HOMEWORK 3
Due by Friday 21. September

Problems

1. The magnetic field of a remote sensing wave propagating through Si (nonmagnetic lossless material) is given by

\[ H = 250 \cos(10^9 t - 5y) \text{ (mA/m)} \] (1)

Find (a) the direction of wave propagation, (b) the phase velocity, (c) the wavelength in the material, (d) the relative permittivity of the material (Hint: think about the relationship between phase velocity, velocity of the light in the air and dielectric constant) and (e) the electric field in phasor form and in a form containing time-dependence.

2*. A satellite communications 1-GHz sinusoidal wave is traveling in the +y direction in a lossless nonmagnetic medium with relative permittivity \( \varepsilon_r = 9 \). The electric field is polarized along the x-direction (contains only an x-component), its peak value is 3 V/m and its initial phase is 0. Write the general expressions for the electric and magnetic fields in phasor form and in a form containing time-dependence.

3. The electric field phasor of a uniform plane wave used for mine detection is given by

\[ \hat{E} = \hat{\jmath} 10 e^{j0.2z} \text{ (V/m)} \]. If the phase velocity of the wave is \( 1.5 \times 10^8 \text{ (m/sec)} \) and the relative permeability of the medium is \( \mu_r = 2.4 \), find (a) the wavelength, (b) the frequency \( f \) of the wave, (c) the relative permittivity of the medium (Hint: for a magnetic medium, the phase velocity is \( u_p = c_o/(\mu_r \varepsilon_r) \)) and (d) the magnetic field \( \mathbf{H}(z,t) \). BE CAREFUL WITH THE PROPAGATION DIRECTION OF THE WAVE!!!

4.* The electric field of a radiotelescope plane wave propagating in a nonmagnetic material is given by

\[ E = [\hat{\jmath} 3 \sin(2\pi 10^7 t - 0.4\pi x) + \hat{x} 4 \cos(2\pi 10^7 - 0.4\pi x)] \text{ (V/m)} \] (2)

Determine (a) the wavelength, (b) the dielectric constant of the material \( \varepsilon_r \) and (c) the magnetic field \( \mathbf{H}(x,t) \).

5. A wireless communication antenna is transmitting waves, described by the electric field:

\[ E(z,t) = \hat{x}|E_{xo}| \cos(\omega t - kz) + \hat{y}|E_{yo}| \cos(\omega t - kz + \delta), \] (3)

Identify the polarization state (Linear, RHS/LHS Circular, ...) and sketch the locus of \( E(0,t) \) for each of the following cases:
(a) \( |E_{xo}| = 3V/m, |E_{yo}| = 4V/m, \delta = 0^\circ \),
(b) \( |E_{xo}| = 3V/m, |E_{yo}| = 4V/m, \delta = 180^\circ \),
(c) \( |E_{xo}| = 3V/m, |E_{yo}| = 3V/m, \delta = 90^\circ \),
(d) \( |E_{xo}| = 3V/m, |E_{yo}| = 3V/m, \delta = -90^\circ \),
(e) \( |E_{xo}| = 3V/m, |E_{yo}| = 4V/m, \delta = -45^\circ \).
6. Repeat Question (5.) for waves of the form

\[ E(y, t) = \hat{x}|E_xo|\cos(\omega t - ky) + \hat{z}|E_{zo}|\cos(\omega t - ky + \delta), \]  

and substituting \(|E_{zo}|\) for the \(|E_{yo}|\) in cases (a)-(d).

7. The electric field of a uniform plane wave propagating in free space is given by \( \tilde{E} = (\hat{x} + j\hat{y})20 e^{-j\pi z/6} \ (V/m) \) for a ground-penetrating radar. Specify the modulus, direction and polarization state of this wave. Is a receiving antenna measuring E-fields above 15 V/m going to be pick up the signal?

8*. A linearly polarized plane wave of the form \( \tilde{E} = \hat{x}|E_xo|e^{-jkz} \) can be expressed as the sum of an RHC polarized wave with amplitude \( a_R \) and an LHC polarized wave with amplitude \( a_L \). Prove this statement by finding expressions for \( a_R, a_L \) in terms of \(|E_{xo}|\).

9. An underwater communication signal has the form: \( \tilde{E} = \hat{x}4\cos(\omega t - kz + 30^\circ) - \hat{y}3\sin(\omega t - kz + 60^\circ) \mu V/m \). Find the state of the polarization. (*) Non-graded (Practice) Problems.