Comments on "Numerical Errors in the
Computation of Impedances by FDTD
Method and Ways to Eliminate Them"

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and Linda P. B. Katehi

In the above paper, a new formula for the calculation of the
characteristic impedance of transmission lines, \( Z_0 \), using the FDTD
method has been given. This formula eliminates the numerical errors
due to the offset of voltages and currents in space and time. We have
recalculated the two examples given in the above paper and confirmed
the presented results. However, we would like to point out that using
the exact formula for the characteristic impedance of a stripline [1]
yields a value of \( Z_0 = 50.2 \, \Omega \) for the stripline investigated in the
above paper. Thus, Fang's result for the real part of \( Z_0 \) (see Fig. 3(a)
in the above paper) has an absolute error of about 8.6 \( \Omega \) and a
relative error of \(-17\%\). This is clearly unacceptable for practical
use and one has to choose a much finer discretization to obtain
reasonable results. Figs. 1 and 2 show our results for a discretization
of \( \Delta x = \Delta y = \Delta z = 0.125 \, \text{mm} \). The real and imaginary parts
of the impedance are given in \( \Omega \). In this case, compared to the exact \( Z_0 \),
the absolute and relative errors in the real part are around 1.2 \( \Omega \) and
\(-2.4\%\), respectively. It is clear from the figures that, for this example,
Fang's new formula does not give any significant improvement over
the other formulas for a discretization fine enough to obtain accurate
results. Furthermore, note that in addition to the formulas cited in the
above paper, we have also included the ratio \( V_k(\omega)/I_{k-1}(\omega) \), which
gives better results than the ratio \( V_k(\omega)/I_k(\omega) \).

REFERENCES


Authors' Reply by Jiayuan Fang and Danwei Xue

In our paper, a simple formula was provided to eliminate the numerical error caused by the offsets of voltages and currents in
space and time. Of course, the smaller the space-step of the finite-
difference grid is used, the smaller such an error will be. In response
to Dib et al.'s comments, the variation of the numerical error with
respect to the space-step \( \Delta h \) is presented as follows.

Suppose the voltage \( v^n_k \) and the current \( i^n_k \) are offset by half a
space-step as shown in Fig. 3, and half a time-step as well. Also

\[ V_k(\omega) = \sum_n v^n_k e^{-j\omega_n \Delta t \Delta t} \]

where the half time-step offset between the voltage and the current
is not accounted for. Assume the time-step \( \Delta t \) is chosen to be 0.5
\( \Delta h/v \) in the following discussion, where \( v \) is the speed of light in
the medium. If the current \( i^n_k \) leads the voltage \( v^n_k \) by half a time-step,
then

\[ V_k(\omega)/I_k(\omega) = Z_0(\omega)e^{j(k \Delta h + \omega \Delta t)/2} \approx Z_0(\omega)e^{j\Delta t/4} \]

\[ V_k(\omega)/I_{k-1}(\omega) = Z_0(\omega)e^{-j(k \Delta h - \omega \Delta t)/2} \approx Z_0(\omega)e^{-3j\Delta t/4} \]

Fig. 1. Real part of the characteristic impedance of a stripline.

Fig. 2. Imaginary part of the characteristic impedance of a stripline.

suppose that the Fourier transforms of \( v^n_k \) and \( i^n_k \) are computed by

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4. IEEE Log Number 9414121.
5. Manuscript received June 8, 1995.
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where \( Z_0(\omega) \) is the characteristic impedance without the error due to the spatial and time offsets of the voltage and the current. From (2) and (3), it can be seen that the error in the phase of \( V_k(\omega)/I_k(\omega) \) is linearly proportional to the space-step \( dh \), and is about one third of that in \( V_k(\omega)/I_{k-1}(\omega) \). When \( k \Delta h \) is small, the error in the phase of the impedance will mainly appear as the error in the imaginary part of the impedance. On the other hand, if the current \( i_k^* \) lags the voltage \( v_k^* \) by half a time-step, then one can easily find that the error in the phase of \( V_k(\omega)/I_k(\omega) \) is about three times of that in \( V_k(\omega)/I_{k-1}(\omega) \).

Relative errors in the impedance calculated from (2) and (3) are

\[
\frac{\Delta Z_0}{Z_0} = \left| 1 - e^{\pm j(\omega \Delta t)/2} \right| \quad (4)
\]

\[
\frac{\Delta Z_0}{Z_0} = \left| 1 - e^{\pm j(\omega \Delta t)/2} \right|, \quad (5)
\]

Fig. 4 displays \( |\Delta Z_0/Z_0|_1 \) and \( |\Delta Z_0/Z_0|_2 \) versus \( \lambda/dh \), where \( \lambda \) is the wavelength in the medium.

A general rule of thumb in selecting the space-step \( dh \) in FDTD computations is to ensure that the shortest wavelength \( \lambda \) of interest is about \( 10 \sim 20 \, dh \). As can be seen from Fig. 4, when \( \lambda = 10 \, dh \), \(|\Delta Z_0/Z_0|_1 \) and \(|\Delta Z_0/Z_0|_2 \) are about 16 and 47\%; and when \( \lambda = 20 \, dh \), the errors expressed by \(|\Delta Z_0/Z_0|_1 \) and \(|\Delta Z_0/Z_0|_2 \) are about 8 and 24\%. In our paper, \( dh = 1 \, mm \) is used, which corresponds to \( \lambda = 10 \, dh \) at the highest frequency of concern at 15 GHz. In the comments, the space-step \( dh \) is reduced to 0.125 mm, which corresponds to \( \lambda = 80 \, dh \) at \( f = 15 \) GHz. Relative errors in the impedances, computed from (4) and (5), are 2.0 and 5.9\%, respectively, at \( \lambda = 80 \, dh \), which are consistent with those shown in Fig. 2. This level of error may still not be considered negligible even with such a fine grid.

As can be clearly seen from Fig. 4, the numerical error caused by just the space and time offsets in voltages and currents can be quite significant. This error mostly appears in the imaginary part and in the frequency-dependent behaviors of both the real and imaginary parts of the calculated impedance. Although one of the expressions in (2) and (3) gives smaller error than the other one, the simple formula provided in our paper completely eliminates such an error.

The other source of the numerical error in calculating the impedance mainly comes from the loss of field singularity at the edges of metal strips in the FDTD formulation. This error will

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