# A Novel Time-Domain Technique for the Analysis of MEMS-Based Variable Capacitors with Moving Metallic Parts

Michiko Kuroda[1], Noriyuki Miura[1], Manos M. Tentzeris[2]

[1]School of Engineering, Tokyo University of Technology, 1404-1 Katakura, Hachioji, Tokyo 192-0982, Japan
Phone; +81-426-37-2498,Fax;+81-426-37-2498, E-mail:kuroda@cc.teu.ac.jp
[2]School of ECE, Georgia Institute of Technology, 777 Atlantic Drive, Atlanta, GA , 30332-250, U.S.A.

***Abstract*** **A new numerical approach is proposed for the analysis of the MEMS-based variable devices with moving boundaries using body fitted grid generation method. Recently, MEMS technology is growing rapidly in RF field and the accurate design of RF MEMS switches, that can be used for phase shifting or reconfigurable tuners, require the computationally effective modeling of their transient and steady-state behavior including the accurate analysis of their time-dependent moving boundaries. Due to the limitations of the conventional numerical techniques, it is tedious to simulate these problems numerically. This new technique proposed in this paper is based on the finite-difference time-domain method with an adaptive implementation of grid generation. Employing this transformation, it is possible to apply the grid generation technique to the analysis of geometries with time-changing boundary conditions. With such a grid, the FD-TD method can be easily solved using a time-invariant square grid regardless of the shape and the motion of the physical region. Using this new technique, a variable capacitor that consists of two moving plates is analyzed. The numerical results between the velocity of the plates and the capacitance are shown and the transient effect is accurately modeled.**

*Key words*: **moving boundary problem, FD-TD method, MEMS-based variable device**

## 1. Introduction

The study of the electromagnetic field for a moving or a rotating body is an important problem for the realization of new optical devices or microwave devices, such as the RF-MEMS structures used in phase-shifters, couplers or filters [1],[2]. Due to the limitations of the conventional numerical techniques, it is difficult to simulate these problems numerically. In this paper, we propose a new numerical approach for the analysis of the electromagnetic field from a moving or a rotating body[3]-[7]. Employing the transformation with the time factor, it would be possible to apply the grid generation technique of [8] to the time-domain analysis of the moving object. With such grid, the FD-TD method can be solved very easily on a “static” (time-invariant) rectangular grid regardless of the shape and the motion of the physical region. In the past this technique has been applied to the Poisson’s equation, the Laplace’s equation and the Navier-Stokes equation for the flow and heat transfer problems and for preliminary electromagnetic problems. In this paper, this simulation method is applied to the analysis of a two-dimensional MEMS variable capacitor.

## 2. Two- Dimensional Variable Capacitor

Without loss of generality, the geometry to be analyzed is shown in Fig.1. The variable capacitor is assumed to move with the constant velocity for the x-direction under the combined effect of mechanical (spring) and electrical forces. For the two-dimensional TM-propagation case, there are only $E_x$, $E_y$, $H_z$ nonzero components with a time variation given by the following equations:

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \tag{1}$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - J_x\right) \tag{2}$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial x} + J_y\right) \qquad (3)$$

where $\varepsilon$, $\mu$ are the constitutive parameters of respective medium. The configuration of the physical and of the computational regions is shown in Fig.1. The interdigitated fingers are assumed to move to the opposite directions of the x-axis with the same velocity v. A positive v means that the conductor moves to the positive x direction. Using a coordinates' transformation technique, the time-changing physical region (x,y,t) can evolve to a time-invariant computational domain. For the geometry of Fig.1, the transform equations between the physical and the computational regions are chosen as follows:

$$\xi = \frac{x - h_n(t)}{h_{n+1}(t) - h_n(t)} \qquad (4)$$

$$\eta = \frac{y - y_n}{y_{n+1} - y_n} \qquad (5)$$

$$\tau = t \qquad (6)$$

$$h_1(t) = x_1 - vt \qquad (7)$$

$$h_2(t) = x_2 + vt \qquad (8)$$

$$h_3(t) = x_3 - vt \qquad (9)$$

$$h_4(t) = x_4 + vt \qquad (10)$$

The functions $h_1(t), h_2(t), h_3(t), h_4(t)$ describe the movement along the x-axis and allow for the realization of a rectangular grid with stationary boundary conditions. According to the relativity postulate, this approach can be easily generalized up to velocities close to the velocity of light adding the appropriate time component. The partial time-derivatives in the transformed domain (ξ,η,τ) can be expressed in term of the partial derivatives of the original domain (x,y,t) using eqs.(4)-(10). The FDTD technique can provide the time-domain solution of the rectangular (ξ,η,τ) grid. The time-step in this case is given by the stability criterion $c\Delta t \le \delta/\sqrt{2}$, for a uniform cell size to x- and y-directions $\delta = \Delta x_0 = \Delta y_0$, where $\delta$ is determined by the finest details of the geometry and is usually smaller than $\lambda/20$.

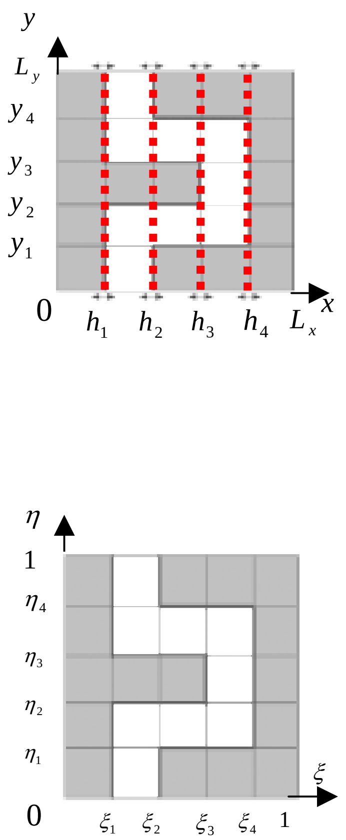


**Fig.1** Physical region and computational region

## 3. Numerical Results

To validate the proposed simulation approach, numerical results were calculated for a two-dimensional variable capacitor with a grid size of 200x200 cells and $L_x = L_y = L = 5\lambda$, $\Delta x = \Delta y = L/200$, $\Delta t = L/800c$. The initial plate separation was $L/5$ and the grid was terminated with absorbing boundary conditions. The relation between the velocity and the transient value of the capacitance between the moving fingers for the 15$^{th}$-time step is shown in Fig 2. It can be observed that different velocity values lead to different values of the capacitance, since they affect the spacing of the fingers for a specific time-step.

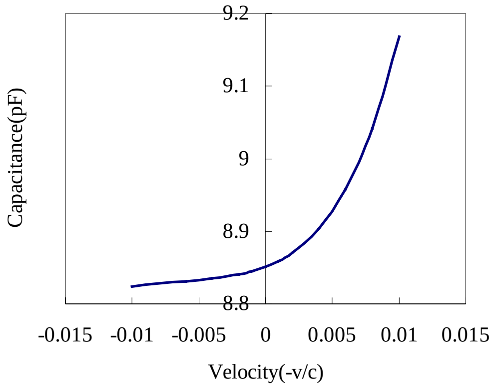


**Fig.2** Capacitance vs Velocity

Following this approach for the whole period of the motion of the fingers, it is easy to perform an accurate analysis of the transient response of the structure and predict the ringing parasitic effects. Fig.3 displays computational results of the time dependence of the transient capacitance for velocity

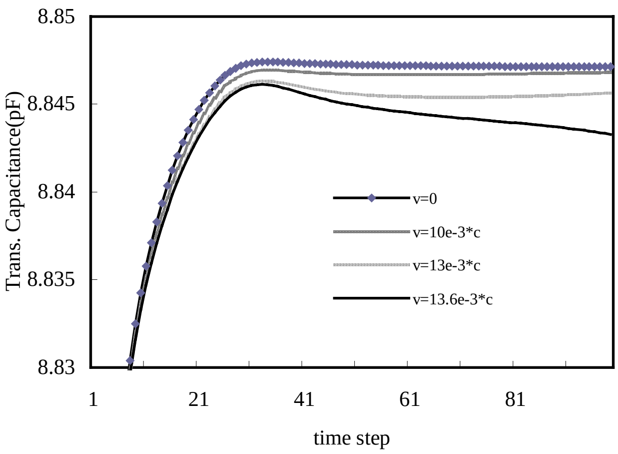


**Fig.3** Time dependence of transient capacitance for each velocity

values in the range of $v = 10.0 \times 10^{-3} c$ to $v = 13.6 \times 10^{-3} c$, assuming that the plates move away from each other from the 1st to the 50th time-step. The stationary value (v=0) is displayed for reference reasons and demonstrates a (smoother) time-variation due to the time evolution of the excitation function itself. It is clear that the transient effect is more pronounced for the higher values of velocity.

## 4. Conclusion

A novel time-domain modeling technique for the analysis of RF structures with moving boundaries has been proposed and applied to the transient analysis of a MEMS-based variable capacitor. This technique is a combination of the FDTD method and the body fitted grid generation technique. The key point of this approach is the enhancement of a space and a time transformation factor that leads to the development of a time-invariant numerical grid. The numerical results from the application of the proposed technique to the analysis of the MEMS capacitor demonstrate its unique computational advantages in the modeling of microwave devices with moving boundaries.


## Acknowledgements

The authors wish to acknowledge the support of the NSF Career Award under #9984761, the Yamacraw Design Center of the State of Georgia, the Georgia Tech Packaging Research Center and Grant-in-Aid for Scientific Research ((c) No.13650435) of The Ministry of Education , Culture, Sports Science and Technology(MEXT), Japan.


## Reference


[1] Aleksander Dec et al., “Microwave MEMS-based voltage-controlled oscillators”, IEEE Trans. MTT, vol.48, No.11, pp.1943-1949, Nov.2000.

[2] N. Bushvager , B. McGarvey, M. Tentzeris, “Adaptive numerical modeling of RF structures requiring the coupling sof Maxwell’s mechanical and solid-state equations”, Proc.of the 2001 ACES, pp.1-8, Monterey, CA, March 2001.

[3] S. Kuroda, H. Ohba, “Numerical analysis of flow around a rotating square cylinder”, JSME International Journal, 36-4 B, pp.592-597, 1993.

[4] M. Kuroda, “Electromagnetic wave scattering from perfectly conducting moving boundary- An application of

the body fitted grid generation with moving boundary", IEICE Trans, Vol.E77-C, No.11, pp1735-1739. , Nov. 1994.

[5] M. Kuroda and S. Kuroda, "The FD-TD method for the analysis of electromagnetic wave scattering from an object moving parallel to the incident wave", Electromagnetic Waves and Electronic Systems(EW&ES), vol.5, No.1, pp.24-31, May 2000.

[6] M. Kuroda, S. Kuroda , "An application of body fitted grid generation method with moving boundaries to solve the electromagnetic field in a moving boundary ", Proc. of the 2001 ACES, pp.519-524, Monterey, CA, March 2001.

[7] M. Kuroda, K. Kawano, M. M. Tentzeris, "Body fitted grid generation method with moving boundaries and its application for analysis of MEMS devices ", Proc. of the 2002 ACES, pp.219-224, Monterey, CA, March 2002.

[8] J. F. Thompson, "Numerical grid generation", North Holland, Amsterdam, 1985.

[9] V. Bladel, "Relativity and engineering", Springer-Verlag, Berlin, 1984.