Body Fitted Grid Generation Method with Moving Boundaries and Its Application for analysis of MEMS

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Abstract
Recently MEMS technology is growing rapidly in RF field. More detailed and complex information is required for a realization of MEMS devices, but the numerical simulation of these moving devices are difficult. In this paper, a new numerical approach to analyze these MEMS devices using body fitted grid generation method with moving boundaries is proposed. This technique is based on the finite-difference time-domain (FD-TD) method and a kind of grid generation. The key feature of this method is the time factor is added to the conventional numerical grid generation. After describing the general theory of body-fitted grid generation method with moving boundaries, the numerical method to solve two-dimensional variable capacitor is proposed.

1. Introduction

The study of the electromagnetic field for a moving or a rotating body is an important problem not only for the electromagnetic probing of moving bodies but also for a realization of new optical devices or microwave devices. Recently more detailed and complex information is required for a realization of MEMS devices, couplers or filters. It becomes necessary to simulate the electromagnetic field distribution in moving conductors. We proposed a new numerical approach for the analysis of the electromagnetic field from a moving or a rotating body. Employing the transformation with time factor, it would be able to apply the grid generation technique of [1] to analyze the moving object. With such grid, the FD-TD method can be solved very easily on a square grid in a rectangular computational region regardless of the shape and the motion of the physical region. We have already applied this technique to the Poisson's equation, the Laplace's equation and the Navier-Stokes equation for the flow and the heat transfer problems and the electromagnetic problems [2]-[6]. In this paper, at first, the general theory of body-fitted grid generation method with moving boundaries is described. Using these methods, the electromagnetic field by a moving dielectric body is analyzed.

2. General Theory of Body-Fitted Grid Generation Method with Moving Boundaries

We have improved the grid generation of [1] to the present one having a coordinate line coincident with arbitrarily shaped moving boundaries or moving bodies. By using this technique, the FD-TD method can be solved very easily on a square grid in a rectangular computational region regardless of the shape and the configuration of the physical region. Even when the boundary shape is changing from time to time, we can solve all these cases in a fixed rectangular coordinate system. Employing the transformation with the time factor, the partial differential equation in the physical region (x, y, z, t) is related to the computational region $(\xi, \eta, \varsigma, \tau)$ as follows:

$$x = x(\xi, \eta, \varsigma, \tau) \tag{1}$$

$$y = y(\xi, \eta, \varsigma, \tau) \tag{2}$$

$$z = z(\xi, \eta, \varsigma, \tau) \tag{3}$$

$$t = t(\xi, \eta, \varsigma, \tau) \tag{4}$$

The inverse transformation is given by

$$\xi = \xi(x, y, z, t) \tag{5}$$

$$\eta = \eta(x, y, z, t) \tag{6}$$

$$\varsigma = \varsigma(x, y, z, t) \tag{7}$$

$$\tau = \tau(x, y, z, t) \tag{8}$$

According to the transformation, the first derivatives are transformed as follows,

$$\begin{bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \\ \partial/\partial t \end{bmatrix} = K \begin{bmatrix} \partial/\partial \xi \\ \partial/\partial \eta \\ \partial/\partial \varsigma \\ \partial/\partial \tau \end{bmatrix} \tag{9}$$

The inverse transformation is given by,

$$\begin{bmatrix} \partial/\partial\xi \\ \partial/\partial\eta \\ \partial/\partial\varsigma \\ \partial/\partial\tau \end{bmatrix} = L \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \\ \partial/\partial t \end{bmatrix} \tag{10}$$

where the matrices K and L are given by

$$K = \begin{bmatrix} \partial\xi/\partial x & \partial\eta/\partial x & \partial\varsigma/\partial x & \partial\tau/\partial x \\ \partial\xi/\partial y & \partial\eta/\partial y & \partial\varsigma/\partial y & \partial\tau/\partial y \\ \partial\xi/\partial z & \partial\eta/\partial z & \partial\varsigma/\partial z & \partial\tau/\partial z \\ \partial\xi/\partial t & \partial\eta/\partial t & \partial\varsigma/\partial t & \partial\tau/\partial t \end{bmatrix} \tag{11}$$

and

$$L = K^{-1} = \begin{bmatrix} \partial x/\partial\xi & \partial y/\partial\xi & \partial z/\partial\xi & \partial t/\partial\xi \\ \partial x/\partial\eta & \partial y/\partial\eta & \partial z/\partial\eta & \partial t/\partial\eta \\ \partial x/\partial\varsigma & \partial y/\partial\varsigma & \partial z/\partial\varsigma & \partial t/\partial\varsigma \\ \partial x/\partial\tau & \partial y/\partial\tau & \partial z/\partial\tau & \partial t/\partial\tau \end{bmatrix} \tag{12}$$

The normal derivatives used as boundary conditions are written as follows.

$$\frac{\partial}{\partial n} = l_x\left(\frac{\partial}{\partial x}\right) + l_y\left(\frac{\partial}{\partial y}\right) + l_z\left(\frac{\partial}{\partial z}\right) = c_1\left(\frac{\partial}{\partial\xi}\right) + c_2\left(\frac{\partial}{\partial\eta}\right) + c_3\left(\frac{\partial}{\partial\varsigma}\right) \tag{13}$$

here l_x, l_y, l_z are direction cosines on the boundary. By this transformation, there is an unique correspondence between the computational region and the physical region. The transformed region can be easily solved in the rectangular computational region by FD-TD method.

3. Two- dimensional Tunable Capacitor

3.1 Problem Configuration and Model

The geometry that will be considered here is shown in Fig.1. By mechanical and electrical force, the tunable capacitor is assumed to move for the x-direction. The incident wave propagates in the x-axis. For the two-dimensional TM-propagation case, as shown in Fig.1, there are only Ex, Ey, Hz nonzero components. The problem is to solve the following Maxwell's equations.

$$\frac{\partial H_z}{\partial t} = c\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \tag{14}$$

$$\frac{\partial E_x}{\partial t} = \frac{c}{n^2}\left(\frac{\partial H_z}{\partial y} - J_x\right) \tag{15}$$

$$\frac{\partial E_y}{\partial t} = -\frac{c}{n^2}\left(\frac{\partial H_z}{\partial x} + J_y\right) \tag{16}$$

$$\xi = \frac{x - h_n(t)}{h_{n+1}(t) - h_n(t)} \tag{17}$$

$$\eta = \frac{y - y_n}{y_{n+1} - y_n} \tag{18}$$

$$\tau = t \tag{19}$$

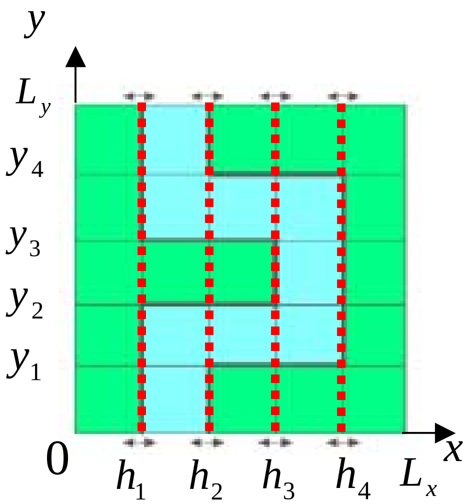
$$h_1(t) = x_1 - vt \tag{20}$$

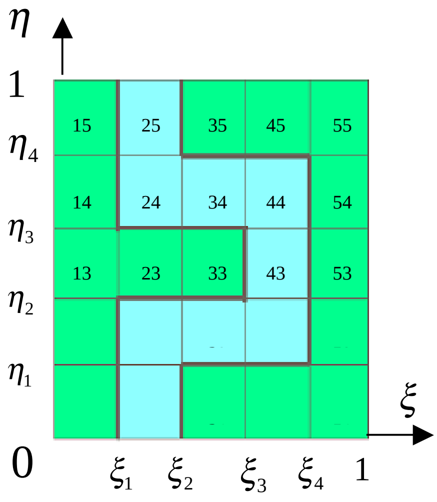
$$h_2(t) = x_2 + vt \tag{21}$$

$$h_3(t) = x_3 - vt \tag{22}$$

$$h_4(t) = x_4 + vt \tag{23}$$

In Fig.1, the configuration of the physical region and the computational region are shown. The physical region (x,y,t) can be normalized by introducing the





computational region, by the following equations. The transform equations between the physical region and the computational region are chosen as follows.

The function $h_1(t), h_2(t), h_3(t), h_4(t)$ shows the movement for x-axis. The problem is to solve the transformed Maxwell's equations. Using Eqs(17)~(23), the partial derivatives with respect to time dependent the curvilinear coordinates are related to the partial derivatives with respect to the cartesian coordinates. In this cartesian coordinate system, it is easy to apply the FD-TD algorithm to solve the equation.. According to the stability criterion, the stability criterion in this case are chosen $c\Delta t \le \delta / \sqrt{2}$, where $\delta = \Delta x_0 = \Delta y_0$, δ is a space increment for x direction when the grid increment is minimum.

3.2 Numerical Results

In this section, numerical results are present. We derived the numerical results of the electromagnetic field passed through the moving dielectric body.
For the simulation of electromagnetic propagation, we assumed the parameters as follows. The grid numbers are $(i, j) = (200,200)$. $L_x = L_y = 1(m)$, $\Delta x = \Delta y = 0.005(m)$, $\Delta t = 4.15 \times 10^{-12}(\sec)$. And the incident wave is input at

Outside of the area are surrounded by the absorbing boundary conditions and the grids in the area.

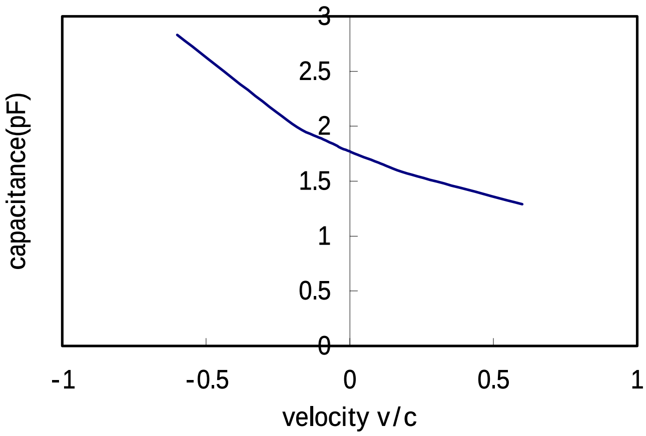


Fig2. Capacitance vs. velocity

The relation between the velocity and the capacitance are shown in Fig 2.
The horizontal axis indicates the velocity and the vertical axis indicates the capacitance. It can be observed the capacitance is dependent on the velocity.

Conclusions

A novel modeling technique, based on the FD-TD method, which can be used to simulate the tunable capaciotor has been described. It is the key point of this method that the time factor is added to the conventional numerical grid generation. We have applied this method to analyze MEMS device, especially , variable capacitor. We believe the technique has many useful features for the moving boundary problems and MEMS devices..

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