A Novel Numerical Approach for the Analysis of 2D MEMS-Based Variable Capacitors with Motion to Arbitrary Directions

Michiko Kuroda(1) , Noriyuki Miura(1) , Manos M. Tentzeris(2)

kuroda@cc.teu.ac.jp

(1) *School of Engineering, Tokyo University of Technology, Hachioji, Tokyo 192-0982, Japan*
(2) *School of ECE, Georgia Institute of Technology, Atlanta, GA 30332-250, USA*

Abstract
A novel time-domain technique is proposed for the analysis of MEMS-based variable devices involving motion to arbitrary in-plane directions using the adaptive body fitted grid generation method with moving boundaries. MEMS technology is growing rapidly in the RF field and the accurate design of RF MEMS switches that can be used for phase shifting or reconfigurable tuners requires the computationally effective modeling of their transient and steady-state behavior including the accurate analysis of their time-dependent moving boundaries. Due to the limitations of the conventional time-domain numerical techniques, it is tedious to simulate these problems numerically. The new technique proposed in this paper is based on the finite-difference time-domain method with an adaptive implementation of grid generation. Employing this transformation, it is possible to apply the grid generation technique to the analysis of geometries with time-changing boundary conditions. A variable capacitor that consists of two metal plates that can move to arbitrary in-plane directions plates is analyzed as a benchmark. The numerical results expressing the relationship between the velocity of the plates and the capacitance are shown and the transient effect is accurately modeled.

1. Introduction

The accurate knowledge of the electromagnetic field variation for a moving or rotating body is very important for the realization of new optical devices or microwave devices, such as the RF-MEMS structures used in phase-shifters, couplers or filters [1,2]. In this paper, we propose a new numerical approach for the analysis of this type of problems that alleviates the limitations of the conventional time-domain techniques[4]-[11]. Employing the transformation with the time factor, it is possible to apply the grid generation technique of [3] to the time-domain analysis of the moving object. With such a grid, the FD-TD method can be solved very easily on a "static" (time-invariant) rectangular mesh regardless of the shape and the motion of the physical region, something that makes it an especially good tool to analyze arbitrary shape and motion. In this paper, this simulation method is applied to the analysis of a two-dimensional MEMS variable capacitor with arbitrary in-plane motions.

2. Two- Dimensional Variable Capacitor with Arbitrary Motions

The geometry to be considered here is shown in Fig.1. Under the combined effect of mechanical and electrical force, the two plates are assumed to move with different velocities to arbitrary in-plane directions. For the two-dimensional TM-propagation case, as shown in Fig.1, there are only Ex, Ey, Hz nonzero components with a time variation given by the following equations.

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \tag{1}$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - J_x\right) \tag{2}$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial x} + J_y\right) \tag{3}$$

where ε , μ are the constitutive parameters of respective medium. In Fig.1, the configurations of the physical and of the computational regions are shown. The interdigitated fingers are assumed to move to arbitrary directions in the *xy*-plane with velocities v and u, respectively and the direction of their motion is shown by the angles θ_v and θ_u. Using a coordinates' transformation technique, the time-changing physical region (x,y,t) can evolve to a time-invariant computational domain. For the geometry of Fig.1, the transform equations between the physical and the computational regions are chosen as :

$$\xi = \frac{x - h_n(t)}{h_{n+1}(t) - h_n(t)} \tag{4}$$

$$\eta = \frac{y - g_m(t)}{g_{m+1}(t) - g_m(t)} \tag{5}$$

$$\tau = t \tag{6}$$

,where n=1,2,3, m=1,2,3 and $h_1(t), h_2(t), h_3(t), h_4(t)$, $g_1(t), g_2(t), g_3(t), g_4(t)$ are written in the following form assuming that the plate velocities remain constant for the whole time of their motion.

$$h_1(t) = x_1 + (v\cos\theta_v)t \tag{7}$$

$$h_2(t) = x_2 + (u\cos\theta_u)t \tag{8}$$

$$h_3(t) = x_3 + (v\cos\theta_v)t \tag{9}$$

$$h_4(t) = x_4 + (u\cos\theta_u)t \tag{10}$$

$$g_1(t) = y_1 + (u\sin\theta_u)t \tag{11}$$

$$g_2(t) = y_2 + (v\sin\theta_v)t \tag{12}$$

$$g_3(t) = y_3 + (v\sin\theta_v)t \tag{13}$$

$$g_4(t) = y_4 + (u\sin\theta_u)t \tag{14}$$

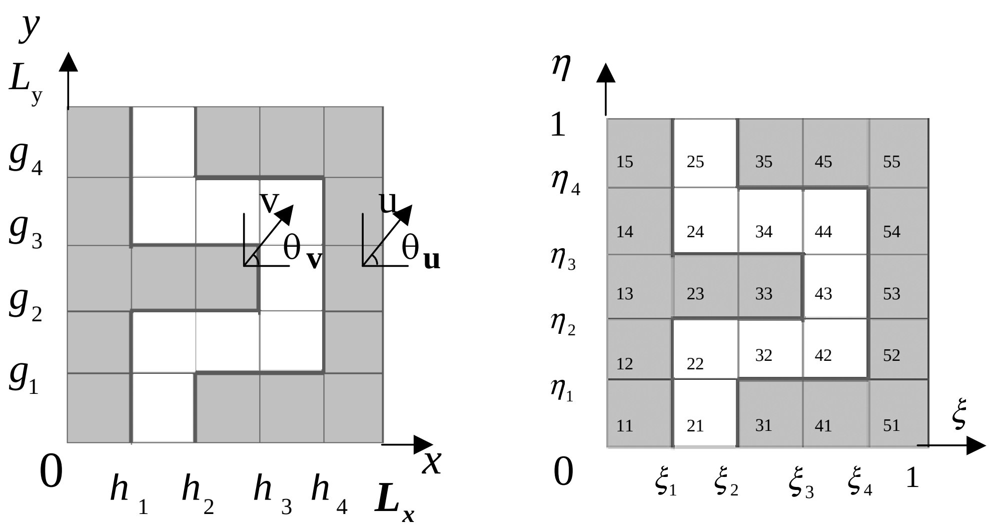

Fig1. Physical region and computational region

The functions $h_1(t), h_2(t), h_3(t), h_4(t)$, $g_1(t), g_2(t), g_3(t), g_4(t)$ describe the movement along the *x* and *y* axis, respectively, and allow for the realization of a rectangular grid with stationary boundary conditions. The partial time-derivatives in the transformed domain (ξ,η,τ) can be expressed in terms of the partial derivatives of the original domain (x,y,t) using eqs.(4)-(14). The FDTD technique can provide the time-domain solution of the rectangular (ξ,η,τ) grid. The stability criterion in this case is chosen as $c\Delta t \le \delta / \sqrt{2}$, where $\delta = \Delta x_0 = \Delta y_0$, assuming the grid is uniformly discretized in both directions. In general, δ is a space increment for *x* and *y* direction when the grid increment is minimum (minimum cell size).

3. Numerical Results

To validate this approach, numerical results are calculated for a two-dimensional variable capacitor with the movement of the fingers only to the *x*-direction. The grid includes 200x200 cells and, $L_x = L_y = L = 5\lambda$, $\Delta x = \Delta y = L/200$, $\Delta t = L/800c$. In this case, as the plates are moving only to the *x*-direction away from each other, the angles are $\theta_u = 0^\circ, \theta_v = 180^\circ$ and as the plates are approaching each other, the angles are $\theta_u = 180^\circ, \theta_v = 0^\circ$. The initial plate separation is $L/5$ and the grid is terminated with Mur's absorbing boundary conditions. The relation between the velocity and the transient value of the capacitance between the moving fingers, assuming that they start to move away and approach each other at t=40 time-step and stop at t=60 time-step, is shown in Fig 2. It can be observed that different velocity values lead to different values of the capacitance, since they affect the spacing of the fingers for a specific t_o time-step. Fig.3 displays computational results of the time dependence of the transient capacitance for velocity values in the range of $u = v = 2\times10^{-3}c$ to $u = v = 8\times10^{-3}c$, assuming that the plates move away from each other from t=10 time-step to t=60 time-step. The horizontal axis indicates the time expressed in time steps and the vertical axis indicates the value of the transient capacitance. The stationary value (v=u=0) is displayed for reference reasons and demonstrates a (smoother) time-variation due to the time evolution of the excitation function itself. In Fig.4, the time dependence of the transient capacitance is demonstrated for various velocity values, assuming that the plates approach each other from t=20 time-step to t=60 time-step. Following this approach for the whole period of the motion of the fingers, it is easy to perform an accurate analysis of the transient response of the structure and predict the ringing parasitic effects. It is clear that the transient effect is more pronounced for the higher values of velocity.

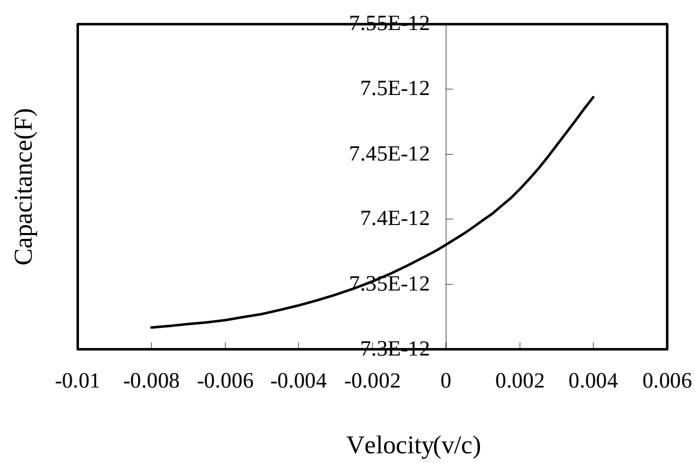

Fig2. Capacitance vs Velocity

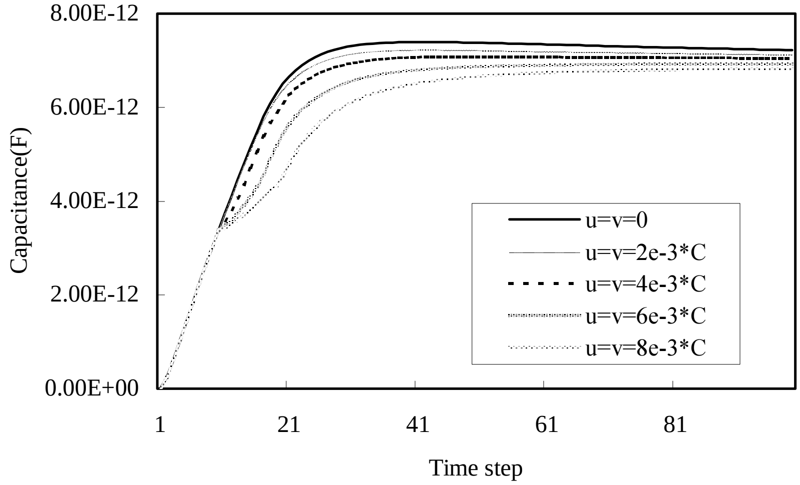

Fig.3 Time dependence of transient capacitance for each velocity, where plates go away from t=10 time- step to t=60 time-step

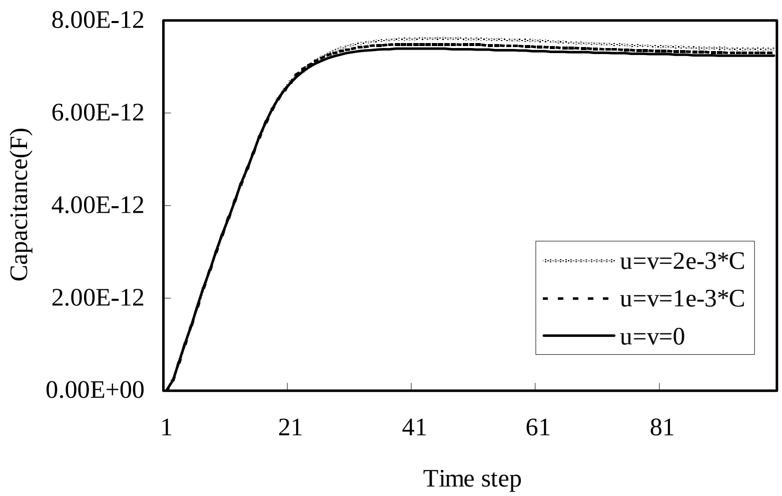

Fig.4 Time dependence of transient capacitance for each velocity, where plates approach each other from t=20 time- step to t= 60 time-step

Conclusions

A novel time-domain modeling technique that has the capability to accurately simulate the transient effect of variable capacitors with arbitrary in-plane motion of their plates has been proposed. This technique is a combination of the FDTD method and the body fitted grid generation technique. The key point of this approach is the enhancement of a space and a time transformation factor that leads to the development of a time-invariant numerical grid. The numerical results of the relation between the capacitance and the velocity of the motion are shown for a MEMS capacitor and demonstrate its unique computational advantages in the modeling of microwave devices with moving boundaries.

Acknowledgements

The authors wish to acknowledge the support of the Georgia Tech NSF Packaging Research Center, the Yamacraw Research Center of the State of Georgia and the NSF CAREER Award #9984761.

Reference

[1] Alekander Dec, et-al, ”Microwave MEMS-Based Voltage-Controlled Oscillators, IEEE Trans MTT, pp.1943-1949, vol.48, No.11, Nov.,2000.

[2] N. Bushvager , B. McGarvey, M. Tentzeris, ”Adaptive Numerical Modeling of RF Structures requiring the Coupling of Maxwell’s mechanical and Solid-State Equations”, Proc. 2001 ACES, pp.1-4, May 2001.

[3] J. F. Thompson, "Numerical Grid Generation'', North Holland, Amsterdam, 1985.

[4] S. Kuroda, H. Ohba, "Numerical analysis of flow around a rotating square cylinder'', JSME International Journal, 36-4 B, pp.592-597, 1993.

[5] M. Kuroda, " A dielectric waveguide with moving boundary'', IEICE Trans., Vol.E74, pp.3952-3954, December1991

[6] M. Kuroda, "Electromagnetic wave scattering from perfectly conducting moving boundary- An application of the body fitted grid generation with moving boundary'', IEICE Trans, Vol.E77-C, No.11, pp.1735-1739. , November 1994.

[7] M. Kuroda, S. Kuroda, "FD-TD method for electromagnetic wave scattering from a moving body by using the body fitted grid generation with moving boundary '', ICEAA99, pp.549-552 , September 1999.

[8] M. Kuroda, S. Kuroda , ”An Application of Body Fitted Grid Generation Method with Moving Boundaries to Solve the Electromagnetic Field in a Moving Boundary ”, Proc. of the 2001 ACES, pp.519-524, March 2001

[9] M. Kuroda, N. Miura, R. Takahashi, ”Numerical Analysis of a Variable Slab Using Body Fitted Grid Generation Method with Moving Boundaries”, PIERS2002, July, 2002

[10] M. Kuroda et al., “Body Fitted Grid Generation Method with Moving Boundaries and Its Application for Analysis of MEMS Devices”, Proc. 2002 ACES, pp.219-224, March 2002.

[11] M. Kuroda, N. Miura, M.M. Tentzeris, ”A Novel Time-Domain Technique for the Analysis of MEMS-Based Variable Capacitors with Moving Metallic Parts ”, Proc. of APMC2002, pp.1208-1211, November 2002.

[12]V. Bladel, ``Relativity and Engineering'', Springer-Verg, Berlin, 1984.