Practical Considerations in the MRTD Modeling of Microwave Structures

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Abstract: This paper focuses on details often overlooked in papers discussing the multiresolution time-domain (MRTD) method. Specifically, a non-uniform Cartesian grid and a uniaxial perfectly matched layer (UPML) implementation for arbitrary levels of wavelet resolution are discussed. The details of implementing the non-uniform grid are simple but non-trivial, allowing a variation of grid size that maintains the orthogonality of the wavelet basis functions. The discussion of the UPML demonstrates issues that arise for arbitrary wavelet resolution UPML and the resulting complexity of the code for arbitrary wavelet functions. An example is presented that compares the Haar-MRTD method with finite-difference time-domain (FDTD) when both UPML and a variable grid are used.

I. Introduction

The multiresolution time-domain (MRTD) technique [1] is often viewed as a generalization of the FDTD method. The method is derived through the application of the method of moments to Ampere's and Faraday's laws in differential form when the electric and magnetic fields and fluxes are discretized using wavelet basis functions. The characteristics of the resulting scheme obviously depend on the wavelet basis functions chosen, and thus MRTD is actually a family of related techniques. Regardless of the basis functions chosen, there are a number of features common to all MRTD schemes.

The feature that gives the technique its name is multiresolution. The wavelet basis functions used in the technique permit the use of a varying number of wavelet coefficients, which can be altered both as a function of position and time. For each level of wavelet resolution the number of basis functions is doubled, and, when implemented correctly, the resolution is also doubled. The term resolution is used in this case to refer to the number of functions (the lowest level function used in the expansion) required per wavelength to provide a certain level of error. The number of these functions required per wavelength is dependent on the basis functions chosen; 10 or more per wavelength are required for Haar basis functions (the simplest wavelets possible) while only slightly more than the Nyquist limit are required for higher order basis functions such as Battle-Lemarie [2].

A number of papers have been written outlining the advantages of the multiresolution time-domain technique for a variety of wavelet bases, demonstrating both a reduction in the number of coefficients required and a decrease in execution time. In this paper two practical elements that are often overlooked in MRTD publications are addressed. The first of these techniques is the implementation of the UPML for arbitrary levels of wavelet resolution. Most papers discussing the UPML show solutions for scaling functions only, and therefore overlook a large increase in computational complexity that can result when wavelet functions are used. The second topic that is addressed is non-uniform gridding. Similar to the non-uniform grid used in FDTD [3], it is often desirable to change the grid size to closely model features of the grid. A simple modification to the existing algorithm can be derived if each axis is modeled individually. However, if this extension to the MRTD algorithm is not performed correctly, the orthogonality of the basis functions is lost, and the scheme becomes implicit.

II. MRTD Background

Many papers have been written discussing MRTD (for example [1,2,4,5]) and as such the derivation of the method will be omitted to conserve space. The notation used in this paper is based on that first presented in [4], with some modifications to generalize it for arbitrary basis functions. The MRTD update equations used in this paper take the form of,

$$ {}_{n+1/2}\mathbf{B}_{x,i,j,k} = {}_{n-1/2}\mathbf{B}_{x,i,j,k} + \frac{\Delta t}{\Delta x \Delta y \Delta z}\left[\sum_m \mathbf{U}^{B_x}_{E_y,m\,n}\mathbf{E}_{y,i,j,k+m} + \sum_m \mathbf{U}^{B_x}_{E_z,m\,n}\mathbf{E}_{z,i,j+m,k}\right], \tag{1} $$

The sum of fields in (1) for the E fields comes from the offset between the E and B fields (and indeed, all components of E and B). The sum is only in one direction because the direction of differentiation in Faraday's and Ampere's laws corresponds with the direction of the field offset. The offset used in the scheme is,

$$ s_d = \frac{\Delta_d}{2^{r_{d,\max}+2}}, \tag{2} $$

where the offset depends on the maximum resolution used in each direction, $r_{d,max}$, and the field component. If the grid points are defined as $(i,j,k)=(i\Delta x, j\Delta y, k\Delta z)$, the electric field components are offset by s_d in their coordinate direction while the magnetic fields are offset in the two directions normal to their coordinate direction.

III. Arbitrary Resolution UPML

The UPML in MRTD can be derived using the same procedure that is used in FDTD [6]. In this case the relationship between the D and H fields can be expressed as

$$ \begin{bmatrix} \dfrac{\partial H_z}{\partial y} - \dfrac{\partial H_y}{\partial z} \\ \dfrac{\partial H_x}{\partial z} - \dfrac{\partial H_z}{\partial x} \\ \dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} \end{bmatrix} = \begin{bmatrix} k_y & 0 & 0 \\ 0 & k_z & 0 \\ 0 & 0 & k_z \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} + \frac{1}{\varepsilon} \begin{bmatrix} \sigma_y & 0 & 0 \\ 0 & \sigma_z & 0 \\ 0 & 0 & \sigma_z \end{bmatrix} \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix}, \tag{3} $$

and the parameters σ and k are varied in the same method as FDTD. A difficulty arises in the calculation of the **U** matricies such as those in (1) because σ and k vary with position. When the inner product integrals are calculated, the orthogonality relationships are no longer useful. The resulting update equation for the D_x field is,

$$ \begin{aligned} {}_{n+1/2}\mathbf{D}_{x,i,j,k} &= \sum_a \sum_b \sum_c \mathbf{U}^{D_x}_{D_x,a,b,c}\,{}_{n-1/2}\mathbf{D}_{x,i+a,j+b,k+c} \\ &\quad + \frac{\Delta t}{\Delta x \Delta y \Delta z}\left[\begin{aligned} &\sum_a \sum_b \sum_c \mathbf{U}^{D_x}_{H_y,a,b,c}\,{}_{n-1/2}\mathbf{H}_{y,i+a,j+b,k+c} \\ &+ \sum_a \sum_b \sum_c \mathbf{U}^{D_x}_{H_z,a,b,c}\,{}_{n-1/2}\mathbf{H}_{z,i+a,j+b,k+c} \end{aligned}\right]. \end{aligned} \tag{4} $$

It is seen that the update depends on the values of the current field and all neighboring fields that overlap with the basis function. A similar relationship exists in the discretization of the constitutive relationship to determine the *E* fields from the *D* fields. If the stencil size of the basis function is 6 cells in each direction, the update of one field component depends on 5184 field components. This significantly increases the computational burden for simulations performed using PML absorbers. However, for the Haar basis functions, where the basis functions do not overlap, the UPML requires the same number of cells as the non-UPML case.

IV Non-Uniform Grid

The MRTD method allows the resolution of each cell to be varied individually, but the resulting equivalent grid points are still equidistantly spaced within the original uniform grid. This method makes it difficult to closely match the features of many common structures. In FDTD, one easy addition to the Yee scheme is to vary the cell spacing as a function of position along each axis individually. This scheme does not change the number or arrangement of grid intersections, only the distance between them. This is also desirable in MRTD, but the offset makes it very important to correctly arrange the grid points so that orthogonality between the grid points is maintained.

This can be accomplished by considering the grid for each field component individually. For example, the E_x components are offset from the fixed Cartesian grid by s_d in the x direction. If Δx is varied as a function of position, then s_d also depends on position. In this case, the value of Δx used for the E_x component at point i is not Δx_i, but $\Delta x_i - s_{d,i} + s_{d,i+1}$. In this fashion, the Δ values used for each component can vary in the same cell, however, the orthogonality between the basis functions is maintained. An easy example can be shown using Haar basis functions.

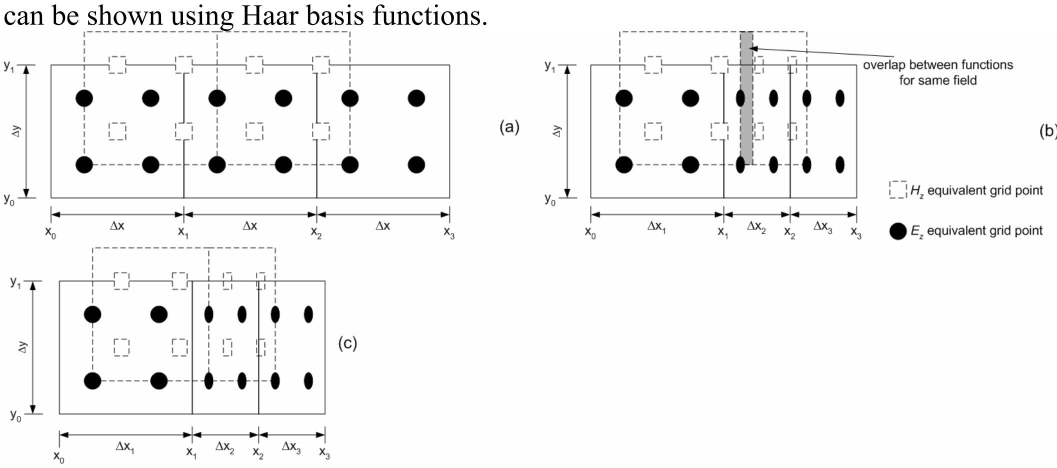

Fig.1: Offset between electric and magnetic fields in MRTD (a) fixed grid (b) non-uniform grid (implemented incorrectly) (c) non-uniform grid (implemented correctly)

In Fig. 1 a Haar MRTD grid demonstrating equivalent grid points for r_{max}=1 is shown in (a). Fig 1 (b) demonstrates the Haar grid if Δx for the E_z component is set the same for each component (to the Δx value used for the grid). In this grid, the basis functions overlap between cells. In a general basis functions this is the same as basis functions for neighboring cells being non-orthogonal. This is fixed in (c), where the cell size is chosen using the criteria given above.

V. Example

To demonstrate the UPML implementation for arbitrary basis functions and the non-uniform grid, a simulation of a microstrip patch antenna was performed using a 3D MRTD code with r_{max}=1. The field offset in this case is 1/8th of a cell width. The return loss results of this antenna simulation are shown in Fig. 2, along with an FDTD simulation of the same antenna. The results are identical, deomonstrating that the PML properly absorbs the fields and that the non-uniform gridding has been implemented in a numerically stable and accurate method. The return loss is high because the antenna has not been optimized for this case. In this example, ε_r is identical above and below the antenna. This was performed because the method of discretizing dielectric interfaces is different between MRTD and FDTD, but the results here are chosen such that the non-uniform grid and UPML are highlighted.

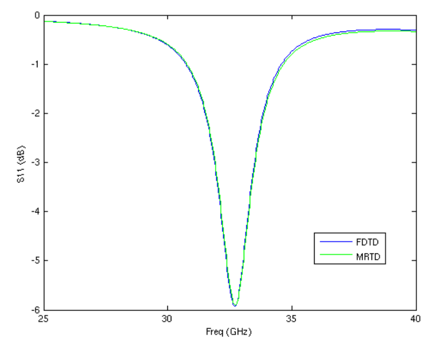

Fig. 2 Return loss of patch antenna, FDTD, MRTD

VI. Conclusions

This paper demonstrates methods that can be used with the MRTD technique that allow the modeling of arbitrary microwave structures. These techniques expand the applicability of the MRTD technique by allowing the UPML to be used for arbitrary wavelet resolution, and allow a non-uniform grid to be correctly applied.

VI. References

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