SUBCELL LUMPED ELEMENT MODELING IN MRTD WITH APPLICATIONS TO METAMATERIAL CHARACTERIZATION

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Abstract: Methods to model metamaterial structures consisting of lumped elements embedded into transmission lines using multiresolution time-domain (MRTD) are presented in this paper. Specifically, this paper presents the modeling of lumped elements in MRTD, including elements that are smaller than a cell (subcell modeling). Using this technique, several lumped elements in any configuration can be modeled within a cell. The technique presented here is based on a deconstruction/reconstruction algorithm originally developed for subcell PEC modeling, and is here extended to lumped elements. MRTD is particularly well suited to metamaterial modeling due to its adaptive gridding. In addition, as a time domain method, properties such as negative group delay can be observed directly in the time domain response.

I. Introduction

The multiresolution time domain technique (MRTD) [1] is an electromagnetic simulation method that allows the modeling of complex structures efficiently using the benefits of multiresolution analysis due to the use of wavelet basis functions. The technique is very similar to other electromagnetic methods in that it is a direct discretization of Maxwell's equations performed in the time domain. The key benefit of MRTD is that the wavelet functions used to discretize these equations can vary resolution as a function of both time and space. This time and space adaptivity allows the resolution to be coarse in areas and times of low field variation. In a practical simulation case, an area of high resolution follows the input pulse, with low resolution used in surrounding areas. By using a thresholding scheme [2] this resolution can be varied automatically with time. While this technique shows significant promise in reducing simulation time while maintaining accuracy, there are several difficulties in applying the technique to complex structures.

Regardless of the basis functions chosen, the discretization of Maxwell's equations used in MRTD follows the same procedure. The method of moments is applied using Galerkin's method and the wavelet functions as basis functions. The resulting equations are an explicit formulation giving the wavelet coefficients for the next time step in terms of the wavelet functions of the previous time step for the surrounding fields. The number of surrounding fields required for the update is dependent on the wavelet functions chosen. The size of the 'cells' used in the MRTD method (a phrase often used because of the similarity between the finite-difference time-domain (FDTD) [3] and MRTD methods) is significantly larger than those used in the Yee-FDTD method. However, the field is not constant in the MRTD cell, and usually varies throughout the cell. The number of values that can occur in the cell is dependent on the wavelet resolution of the cell. This property of MRTD schemes allows the time-and space adaptive gridding; when the field variation across a cell is low, high order wavelet coefficients can be neglected. However, this also causes difficulties in applying localized effects within an MRTD grid, as the coefficients that define a field value at one area can also define field values at neighboring areas. Any modification of wavelet coefficients to apply an effect at one area must be performed so as to not effect the surrounding field values.

Previous work [4] has demonstrated how the use of a wavelet reconstruction/decomposition algorithm can be used with Haar basis functions to apply perfect electrical-conductor (PEC) boundary conditions at the subcell level. This work also demonstrated how, under the correct conditions, MRTD and FDTD simulations are equivalent (if the adaptive gridding of MRTD is not used). Using this technique, a transform is performed to apply pointwise effects to MRTD at the equivalent grid point [5] level. When a PEC is applied to the MRTD scheme, the field values at points tangential to PECs are set to zero. This subcell technique significantly extends the applicability of MRTD because the cell size is not limited by the feature size. However, feature size does dictate the resolution required in areas that intersect PECs. This paper demonstrates and gives an example of how this technique can be used to model lumped elements that are smaller than a single cell in size.

One application of particular interest to this type of modeling is that of metamaterial characterization. These materials, alternatively called negative refractive index (NRI) [6] materials are unique because they have both negative permittivity and permeability (and thus, a negative index of refraction). These materials have a number of unique properties that make them particularly interesting to study. One of these properties is negative group velocity, one consequence of which is that the peak of the output pulse emerges from the material before the peak of input pulse enters. It is important to note that the output pulse is significantly lower and magnitude and is essentially created through the interaction of the wavefront and the resonant structures that make up the metamaterial. These parameters can all be determined by examining both the magnitude and phase of the S-parameters, however, one benefit of using a time-domain simulator is that these characteristics can be observed directly in the time domain response.

This paper demonstrates the modeling of subcell lumped elements in MRTD. In addition to lumped elements, any combination of PECs and material interfaces can be placed within the cell. In this way, this technique allows the modeling of composite cells in MRTD. The methods to apply subcell lumped elements are presented, along with results of simulations using these lumped elements. Finally, the representation of the resonant elements used in a periodically loaded transmission line designed to have a negative refractive index using this technique is presented.

II. MRTD Background

The technique presented in this paper is applied specifically to Haar based MRTD. Haar wavelets are based on pulses and as such have a finite domain and do not overlap with neighboring functions. In addition, they create areas of fixed field value when a finite resolution level is used. These properties allow a direct translation between pointwise values/effects and MRTD based wavelet coefficients. The total number of coefficients required to accurately represent field values is the same as in FDTD, however, the inherent time-and space adaptive grid allows that criterion to be applied subjectively as a function of time and thus fewer operations are required per time step.

To compromise between completeness of presentation and space requirements, a brief 2 dimensional derivation of the MRTD technique is presented. In addition, the dielectric constant is chosen to be constant throughout each cell, removing the step of solving for **E** fields from **D** fields. In a two dimensions, the TE_z mode is represented by,

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\frac{\partial H_z}{\partial y} \tag{1}$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\varepsilon}\frac{\partial H_z}{\partial x} \tag{2}$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left[\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right] \tag{3}$$

the electric field component, $\mathbf{E_x}$, is discretized in a general wavelet basis as

$$E_x(x) = \sum_{n,i,j} h_{n(t)} \cdot \left[\begin{array}{l} {}_nE^{x,\phi\phi}_{i,j}\varphi_i(x)\varphi_j(y) + \sum_{r=0}^{r_{\max}}\sum_{p=0}^{2^r-1} {}_nE^{x,\psi\phi}_{i,j,r,p}\psi^r_{i,p}(x)\varphi_j(y) + \\ \sum_{r=0}^{r_{\max}}\sum_{p=0}^{2^r-1} {}_nE^{x,\phi\psi}_{i,j,r,p}\varphi_i(x)\psi^r_{j,p}(y) + \sum_{r=0}^{r_{\max}}\sum_{p=0}^{2^r-1}\sum_{s=0}^{r_{\max}}\sum_{q=0}^{2^s-1} {}_nE^{x,\psi\psi}_{i,j,r,p,s,q}\psi^r_{i,p}(x)\psi^s_{j,q}(y) \end{array} \right] \tag{4}$$

The four groups of coefficients represent the scaling-*x*/scaling-*y*, wavelet-*x*/scaling-*y*, scaling-*x*/wavelet-*y*, and wavelet-*x*/wavelet-*y* coefficients. The scaling functions are the φ components while the wavelet coefficients are represented as ψ. There are $2^{2(r_{\max}+1)}$ wavelets for a maximum resolution r_{max}. A 3D representation is similar, except there are eight groups of coefficients, representing all of the wavlet/scaling products for all 3 directions.

By applying similar discretizations to the other 3 fields in (1)-(3), and then applying Galerkin's method, update equations for the wavelet/scaling functions are determined. It is convenient to represent the scaling/wavelet coefficients for a single cell as a vector, and thus the update equation for all coefficients in a cell appear as a vector equation. In addition, the inner product terms between the wavelet functions appear as a matrix. Using this representation, the update equation for Haar-MRTD are,

$$_{\mathbf{n}}\mathbf{E}^{\mathbf{x}}_{\mathbf{i,j}} = {}_{\mathbf{n-1}}\mathbf{E}^{\mathbf{x}}_{\mathbf{i,j}} + \frac{\Delta t}{\varepsilon \Delta y}\left(\mathbf{U}_{\mathbf{E_{x1}}}\, {}_{\mathbf{n-1}}\mathbf{H}^{\mathbf{z}}_{\mathbf{i,j}} + \mathbf{U}_{\mathbf{E_{x2}}}\, {}_{\mathbf{n-1}}\mathbf{H}^{\mathbf{z}}_{\mathbf{i,j-1}}\right) \tag{5}$$

$$_{\mathbf{n}}\mathbf{E}^{\mathbf{y}}_{\mathbf{i,j}} = {}_{\mathbf{n-1}}\mathbf{E}^{\mathbf{y}}_{\mathbf{i,j}} - \frac{\Delta t}{\varepsilon \Delta x}\left(\mathbf{U}_{\mathbf{E_{y1}}}\, {}_{\mathbf{n-1}}\mathbf{H}^{\mathbf{z}}_{\mathbf{i,j}} + \mathbf{U}_{\mathbf{E_{y2}}}\, {}_{\mathbf{n-1}}\mathbf{H}^{\mathbf{z}}_{\mathbf{i-1,j}}\right) \tag{6}$$

$$\mathbf{}_{n}\mathbf{H}^{z}_{i,j} = {}_{n-1}\mathbf{H}^{z}_{i,j} + \frac{\Delta t}{\mu}\left(\frac{1}{\Delta y}\left(\mathbf{U}_{H_{Ex1}}\,{}_{n-1}\mathbf{E}^{x}_{i,j} + \mathbf{U}_{H_{Ex2}}\,{}_{n-1}\mathbf{E}^{x}_{i,j+1}\right) - \frac{1}{\Delta x}\left(\mathbf{U}_{H_{Ey1}}\,{}_{n-1}\mathbf{E}^{y}_{i,j} + \mathbf{U}_{H_{Ey2}}\,{}_{n-1}\mathbf{E}^{y}_{i+1,j}\right)\right) \quad (7)$$

These equations are exactly the same as their FDTD counterparts if the vectors are replaced with scalars and the update matricies, **U**, are replaced with coefficients. The equations for other wavelet bases are also very similar, except that the update of each point depends on a larger number of surrounding cells, and thus the **UE** terms are sums.

III. Subcell Lumped Elements in MRTD

In (5)-(7), all wavelet/scaling coefficients for a given cell are updated at once. Applying localized effects becomes difficult because a change in a single coefficient effects the field at several equivalent grid points. In order to apply effects in a pointwise fashion, (5)-(7) can be transformed from wavelet to localized values by multiplying the equation with a reconstruction matrix, **R** [4]. In Haar-MRTD schemes the number of equivalent grid points is equal to the number of wavelet/scaling coefficients, and thus reconstruction is performed by multiplying the coefficient vector with an easily calculated square matrix. When the equations are in a pointwise form, effects that apply to only a single equivalent grid point can be applied, and then the equations can be transformed back to the wavelet form. For example, PECs can be applied by zeroing fields at points tangential to metals. If a general pointwise matrix, **G**, is used, the new update equations are identical to (5)-(7) if the **U** matricies are replaced by $\mathbf{U}^{\mathbf{G}}$ versions where

$$\mathbf{U}^{G} = \mathbf{R}^{-1}\mathbf{G}\mathbf{U}'\mathbf{R} \quad (8)$$

The application of any pointwise effect in the Haar-MRTD scheme becomes a function of using the appropriate **G** matrix. If the pointwise effect that is being applied is truly local (involves no surrounding points), then **G** is a diagonal matrix. It has been shown in FDTD [7] that lumped R, L, or C elements can be represented by making the appropriate extensions to the E field update equations. For example, a term can be added to the $\mathbf{E_z}$ update equation to represent a Z oriented lumped element. In this case

$$_{n+1}E^{z}_{i,j,k} = {}_{n}E^{z}_{i,j,k} + \frac{\Delta t}{\varepsilon_0}\nabla\times {}_{n+\frac{1}{2}}H_{i,j,k} - \frac{\Delta t I_z}{\varepsilon_0 \Delta x \Delta y} \quad (9)$$

for a resistor, $\mathbf{I_z}$ can be represented as

$$I_z = \frac{\Delta z}{2R}\left(E_z^{n+1} + E_z^{n}\right) \quad (10)'$$

and the resulting equation is

$$_{n+1}E^{z}_{i,j,k} = \left(\frac{1-\dfrac{\Delta t \Delta z}{2R\varepsilon_0 \Delta x \Delta y}}{1+\dfrac{\Delta t \Delta z}{2R\varepsilon_0 \Delta x \Delta y}}\right){}_{n}E^{z}_{i,j,k} + \left(\frac{\dfrac{\Delta t}{\varepsilon_0}}{1+\dfrac{\Delta t \Delta z}{2R\varepsilon_0 \Delta x \Delta y}}\right)\nabla\times {}_{n+\frac{1}{2}}H_{i,j,k}\,, \quad (11)$$

To represent the resistor in MRTD, the coeffiecients in (11) are placed in the diagonal of **G** in (8). The remainder of the diagonal is set to 1, representing no change to the update matrix. In this way, a resistor can be placed at any subcell point. Similarly, a capacitor can be represented using the coefficient,

$$G_c = \frac{\Delta t}{1+\dfrac{C\Delta z}{\varepsilon \Delta x \Delta y}} \quad (12)$$

on the **H** field cross product terms,and an inductor can be represented using

$$G_i = -\frac{\Delta z (\Delta t)^2}{\varepsilon L \Delta x \Delta y} \quad (13)$$

on a new term representing the sum of previous E fields.

Fig. 1 shows the return loss of an imbedded resistor in a 50Ω transmission line. This line is 500μm wide on a 200μm substrate with ε_r=3.3. The line is terminated with 5 parallel resistors, each occupying only 1/8th of a cell. It can be seen in this case that the resistor offers a very good match for low frequency but that the reflection increases as frequency increases and the cell size is closer to the wavelength.

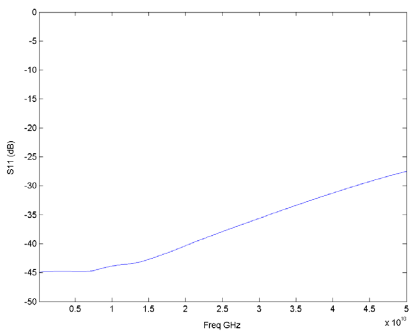

Fig. 1 S_{11} vs. freq

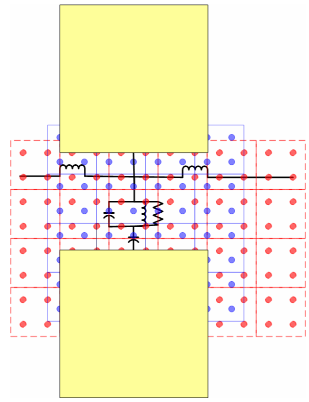

Fig. 2 Lumped elements embedded in MRTD grid

IV. Applications to Loaded Transmission Line Metamaterials

The technique presented above is very useful in the modeling of metamaterials because it allows high resolution grids to be used in areas containing complex circuits. The loaded transmission line metamaterials [6] are often fabricated with chip RLC elements, with very small size compared to the surrounding circuits. Simple circuit simulator characterizations are inaccurate because they cannot calculate a variety of effects that are important for high frequency applications such as radiation loss and substrate modes. Fig. 2 shows a resonating element embedded in a transmission line. The MRTD grid is demonstrated (a 2D cut of a 3D grid showing the Ex and Ez grid points), with the solid and dotted lines showing the Ex and Ez fields, respectively. The equivalent grid points are represented by the dots. Any grid point can be covered by a lumped element, and in this way a resonating element is embedded in the transmission line. Using this representation, the area containing the resonating element can be modeled with high resolution while the surrounding area is modeled with lower resolution cells.

V. Conclusions

This paper demonstrates methods to embed lumped elements into the MRTD grid that allow the modeling of multiple lumped elements per cell. One very interesting application of this work is the simulation of metamaterials consisting of lumped element embedded in transmission lines. Future applications of this work include demonstrating the efficiencies due to adaptive gridding and showing effects such as negative group velocity in the time domain output.

VI. References

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