Optimal Design Parameters for Wireless Power Transfer by Resonance Magnetic

Olutola Jonah, *Student Member, IEEE*, Stavros V. Georgakopoulos, *Senior Member, IEEE*, and
Manos M. Tentzeris, *Fellow, IEEE*

Abstract—The wireless power transfer (WPT) efficiency of systems based on strongly coupled magnetic resonance (SCMR) depends on the Q-factor of the system elements, which is a function of the geometrical parameters. This letter analytically derives the equations that can be used to design optimal loop and helical elements for SCMR systems. Also, for loops and helices, a global maximum condition is derived in order to design SCMR systems with maximal efficiency.

Index Terms—Geometric parameters limits, magnetic resonance, wireless sensor.

I. INTRODUCTION

NUMEROUS wireless power transfer methods have been proposed and studied in the past for various applications. Specifically, wireless power transfer has been achieved using near-field coupling in several applications such as RFID tags, telemetry, and implanted medical devices [1], [2]. In addition, certain inductive coupling techniques have been reported to exhibit high power transfer efficiencies (on the order of 90%) for very short distances (1–3 cm) [3]. However, the efficiency of such techniques drops drastically for longer distance since it decays as $1/r^6$ [4], [5].

This letter focuses on the design of wireless powering systems based on the strongly coupled magnetic resonance (SCMR) method. The SCMR method is a novel nonradiative wireless midrange power transfer method (10–300 cm) that has been recently developed [6]–[11]. In fact, the SCMR method has achieved 40% wireless power transfer efficiency in air for a distance of 2 m for a single receiver [6]. Also, SCMR technique has been used to simultaneously power multiple receivers in air and has achieved 60% at a distance of 2 m [7]. Previous work has shown that SCMR provides wireless power transfer efficiencies that are significantly larger than efficiencies of conventional inductive coupling methods [6], [7], [12]. In order for SCMR to achieve high efficiency, it requires that the
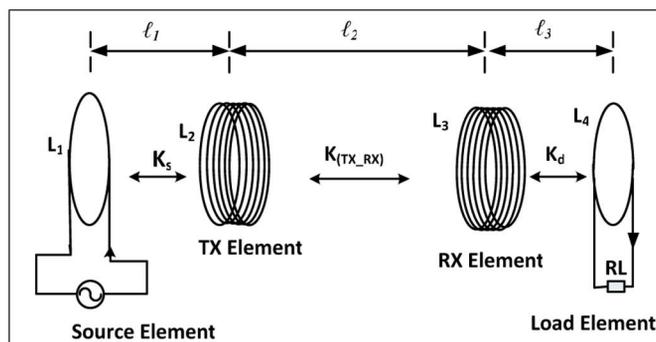
Fig. 1. Schematic of an SCMR power transfer system in air.

transmitting and receiving elements (typically loops or coils) are designed so that they resonate at the desired operational frequency that must coincide with the frequency of where the elements exhibit maximum Q-factor.

This letter analytically derives equations that must be satisfied by the geometrical parameters of loops or helices in order to guarantee optimal efficiency for SCMR systems.

II. WIRELESS POWERING WITH SCMR

SCMR systems use resonant transmitters and receivers that are strongly coupled. Strongly coupled systems are able to transfer energy efficiently because resonant objects exchange energy efficiently versus nonresonant objects that only interact weakly. A standard SCMR system consists of four elements (typically four loops or two loops and two coils) as shown in Fig. 1.

The source element is connected to the power source, and it is inductively coupled to the TX element. The TX element exhibits a resonant frequency that coincides with the frequency, where its Q-factor is naturally maximum. Similarly, the RX exhibits a resonant frequency that coincides with the frequency where its Q-factor is naturally maximum. Furthermore, the load element is terminated to a load. For our analysis, we assume that the entire system operates in air. Also, SCMR requires that the TX and RX elements are resonant at the same frequency in order to achieve efficient wireless power transfer.

III. OPTIMAL SCMR BASED ON LOOPS AND HELICES

In this section, we will derive the guidelines for designing SCMR systems that uses helices as TX and RX elements. The TX and RX elements can be equivalently represented by a series RLC circuit. Helices are often preferred as TX and RX SCMR elements because they exhibit both distributed inductance and

capacitance, and therefore they can be designed to self-tune to a desired resonant frequency f_r without the need of external capacitors. Also, external capacitors have losses, which in practice can reduce the Q-factor of the TX and RX elements and in turn decrease the efficiency of SCMR systems. Based on the equivalent RLC circuit of an SCMR system, its resonant frequency f_r can be calculated by the following equation:

$$f_r = \frac{1}{2\pi\sqrt{LC}}. \tag{1}$$

The resonant frequency f_r is also the operational frequency for the SCMR wireless powering system. The Q-factor of a resonant RLC circuit is given by

$$Q = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R}. \tag{2}$$

Therefore, the Q-factor of a resonant helix (i.e., self resonant) can be written as

$$Q = \frac{2\pi f_r L_{\text{helix}}}{R_{\text{ohm}} + R_{\text{rad}}} \tag{3}$$

where L, R_{rad}, and R_{ohm} are the self-inductance, radiation resistance, and ohmic resistance of the helix, which is for a short helix or solenoid $(2r > h)$ given by [4], [13]–[15]

$$L_{\text{helix}} = \mu_o r N^2 \left[\ln\left(\frac{8r}{r_c}\right) - 2 \right] \tag{4}$$

$$R_{\text{rad}} = \left(\frac{\pi}{6}\right) \eta_o N^2 \left(\frac{2\pi f_r r}{c}\right)^4 \tag{5}$$

$$R_{\text{ohm(helix)}} = \left(\sqrt{\mu_o \rho \pi f_r}\right) \frac{Nr}{r_c} \tag{6}$$

where μ is the permeability of free space, ρ is the helix's material resistivity, r is the radius of the helix, r_c is the cross sectional wire radius, N is the number of turns, f is the frequency, η_o is the impedance of free space, c is the speed of light, and h is the height of the helix. It should also be noted that (3)–(6) are valid only when $r < \lambda/6\pi$ [3].

SCMR requires that both RX and TX helices also exhibit maximum Q-factor at their resonant frequency f_r in order to achieve maximum wireless power efficiency. This can also be seen by the equation for describing the efficiency of an SCMR system derived in at it operation frequency f_r [16] as follows:

$$\eta(f_r) = \frac{k^2_{(\text{TX\_RX})}(f_r) Q_{\text{TX}}(f_r) Q_{\text{RX}}(f_r)}{1 + k^2_{(\text{TX\_RX})}(f_r) Q_{\text{TX}}(f_r) Q_{\text{RX}}(f_r)} \tag{7}$$

where $K_{\text{TX\_RX}}$ is the mutual coupling between the RX and TX helices, whereas Q_{TX} and Q_{RX} are the Q-factors of the RX and TX helices, respectively. If the TX and RX helices are identical their Q-factors are equal i.e., $Q_{\text{TX}} = Q_{\text{RX}} = Q$; therefore, (7) can be written as

$$\eta(f_r) = \frac{k^2_{(\text{TX\_RX})}(f) Q^2_{\text{TX}}(f_r)}{1 + k^2_{(\text{TX\_RX})}(f_r) Q^2_{\text{TX}}(f_r)}. \tag{8}$$

Equation (8) shows that in order to maximize the efficiency of an SMCR system, the operation frequency f_r must be equal

to the frequency f_{max}, where the Q-factor is maximum. In what follows, the maximum Q-factor of a resonant helix is derived. The Q-factor of a resonant helix can be expressed in terms of its geometrical parameters using (3)–(6) as

$$Q(f_r, r, r_c, N) = \frac{2\pi f_r \mu_0 r N^2 \ln\left(\frac{8r}{r_c - 2}\right)}{\left(\frac{\mu_0 \rho \pi f_r r^2 N}{r_c^2}\right)^{1/2} + 20\pi^2 N^2 \left(\frac{2\pi f_r r}{c}\right)^4}. \tag{9}$$

The maximum Q-factor Q_{max} and the frequency f_{max}, where Q_{max} occurs, can be derived from (9) using calculus as

$$f_{\text{max}}(r, r_c, N)$$
$$= \frac{c^{8/7} \mu^{1/7} \rho^{1/7}}{4 \cdot 15^{2/7} N^{2/7} r_c^{2/7} \pi^{11/7} r^{6/7}} \tag{10}$$

$$Q_{\text{max}}(r, r_c, N)$$
$$= \frac{2\pi f_{\text{max}} \mu_0 r N^2 \ln\left(\frac{8r}{r_c - 2}\right)}{\left(\frac{\mu_0 \rho \pi r^2 f_{\text{max}} N}{r_c^2}\right)^{1/2} + 20\pi^2 N^2 \left(\frac{2\pi f_{\text{max}} r}{c}\right)^4}. \tag{11}$$

Based on the above discussion, an SCMR system requires that

$$f_r = f_{\text{max}} \tag{12}$$

which can be written based on (10) as

$$f_r(r, r_c, N) = \frac{c^{8/7} \mu^{1/7} \rho^{1/7}}{4 \cdot 15^{2/7} N^{2/7} r_c^{2/7} \pi^{11/7} r^{6/7}}. \tag{13}$$

Therefore, (13) shows that the geometrical parameters of a helix can be appropriately chosen so that the helix has maximum Q-factor at a chosen frequency f_r. For example, if the parameters f_r, r_c, N, and ρ are specified by a designer, (13) can be solved for the radius of the maximum Q-factor, r_{max}, as follows:

$$r_{\text{max}} = \left[\frac{c^{8/7} \mu^{1/7} \rho^{1/7}}{4 \cdot 15^{2/7} r_c^{2/7} N^{2/7} \pi^{11/7} f_r} \right]^{7/6}. \tag{14}$$

Similar equations to (10), (11), and (13) have been presented in [17] in order to obtain the local maximum for the Q-factor.

Next, the helices are analyzed using (10), (11), and (14) to study the behavior of the maximum Q-factor Q_{max} versus the electrical length of the helix $(C_{\text{dev}}/\lambda_{Q_{\text{max}}})$ at f_{max}, which can be written as

$$\frac{C_{\text{dev}}}{\lambda_{\text{max}}} = \frac{2\pi r_{\text{max}}}{\lambda_{\text{max}}} = \frac{2\pi r_{\text{max}} f_{\text{max}}}{c} \tag{15}$$

where L_{dev} is the length of the helix (device), and λ_{max} is the wavelength corresponding to f_{max} given by (10). Specifically, optimum SCMR helices with $N = 6$ are designed in the frequency range 100 kHz $\leq f \leq$ 5 GHz for four values of the cross-sectional radius, $r_c = 0.01, 0.1, 1.0,$ and 10 mm. The material of the helices is assumed copper, and for each pair of f_{max} and r_c, the optimum r is calculated by (14). Then, Q_{max} from (11) is plotted in Fig. 2 versus the electrical length of the helices $(C_{\text{dev}}/\lambda_{Q_{\text{max}}})$, which is calculated by (15). Specifically, Fig. 2 illustrates that for each pair of f_{max} and r_c, there is an r_{max} that provides the global maximum for the Q-factor, $Q_{G\text{max}}$.
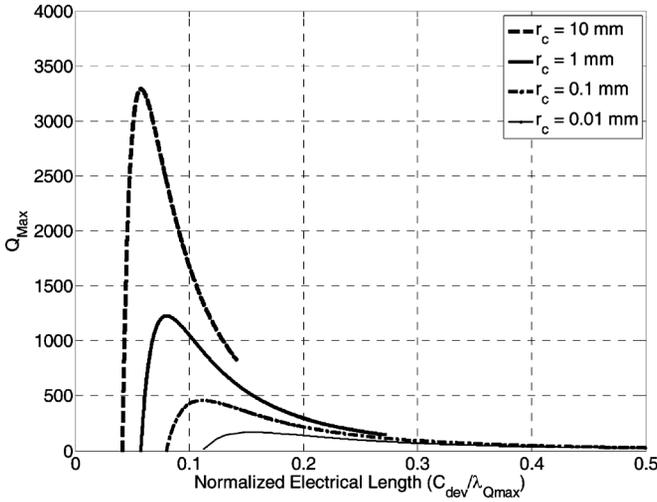
Fig. 2. $Q_{\max}$ versus electrical length of the helix.

Fig. 3. Efficiency of SCMR systems with different r/r_{c} ratios.

In what follows, the global maximum Q-factor of the helix, $Q_{G\max}$, is formulated. First, the local maximum Q-factor, $Q_{L\max}$, is derived by substituting (10) into (11)

$$Q_{L\max} = \frac{2 \cdot 3^{6/7} r_{\mathrm{c}}^{6/7} c^{8/7} \mu^{8/7} N^{6/7} \rho^{1/7} \left[\ln\left(\frac{8r}{r_{\mathrm{c}}}\right) - 2 \right]}{5^{1/7} \pi^{2/7} r^{3/7} \left[c^{4/7} \mu^{4/7} \rho^{4/7} + 6 r_{\mathrm{c}}^{1/7} N^{1/7} r^{3/7} \sqrt{\frac{c^{8/7} \mu^{8/7} \rho^{8/7}}{r_{\mathrm{c}}^{2/7} N^{2/7} r^{6/7}}} \right]}. \tag{16}$$

Using calculus again, we can find out that the global maximum for the Q-factor occurs when

$$\frac{r_{(G\max)}}{r_{\mathrm{c}}} = \frac{e^{13/3}}{8} \approx 9.52. \tag{17}$$

This is an important finding, which shows that the ratio between the helix radius r and the cross-sectional radius r_c must be approximately 9.52 in order to achieve the maximum Q-factor. This ratio is also independent of frequency and material.

Also, by substituting (17) into (16), we can write the global maximum for the Q-factor as

$$Q_{G\max} = \frac{28 \cdot 2^{2/7} r_{\mathrm{c}}^{3/7} c^{8/7} \mu^{8/7} N^{6/7} \rho^{1/7}}{15^{1/7} e^{13/7} \pi^{2/7} \left[c^{4/7} \mu^{4/7} \rho^{4/7} + 6 r_{\mathrm{c}}^{4/7} N^{1/7} \sqrt{\frac{c^{8/7} \mu^{8/7} \rho^{8/7}}{r_{\mathrm{c}}^{8/7} N^{2/7}}} \right]}. \tag{18}$$

Therefore, if a helix is designed to operate at the global maximum Q-factor, it will yield the maximum possible wireless efficiency for the corresponding SCMR system. In order to verify the global maximum design of (17), we assume that an arbitrary ratio of $r/r_{\mathrm{c}} = t$ and solve (13) and (17) to obtain the r and r_{c} given the number of turns, N, and the desired frequency of operation, f_o

$$r_{\mathrm{c}_{\max}} = \frac{c \mu^{1/8} \rho^{1/8}}{2 \cdot 2^{3/4} \cdot 15^{1/4} N^{1/4} f_o^{7/8} \pi^{11/8} t^{3/4}} \tag{19}$$

$$r_{\max} = \frac{c \mu^{1/8} \rho^{1/8} t^{1/4}}{2 \cdot 2^{3/4} \cdot 15^{1/4} N^{1/4} f_o^{7/8} \pi^{11/8}}. \tag{20}$$
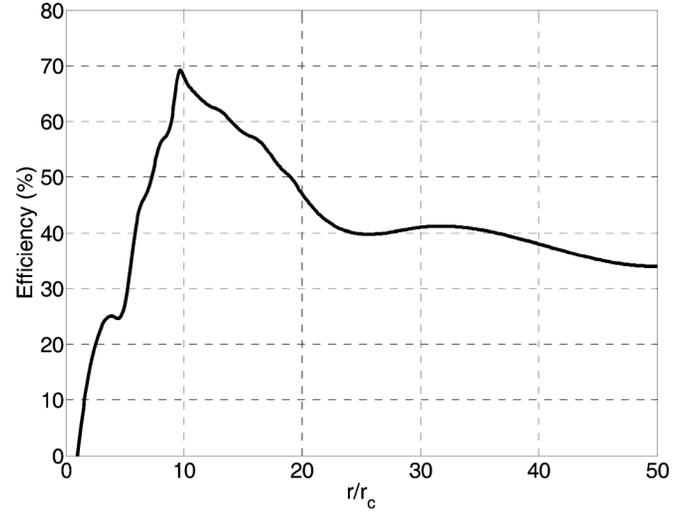
Based on (19) and (20), SCMR systems were designed and simulated in Ansoft HFSS for different ratios $r/r_{\mathrm{c}} (2 \le t \le 50)$ and assuming the number of turns, $N = 5$; distances, $l_1 = l_3 = 2$ cm, $l_2 = 10$ cm (see Fig. 1); and operational frequency, $f_o = 46.5$ MHz. The efficiency of these designs is compared in Fig. 3. The results clearly illustrate that the maximum efficiency is achieved for a ratio of $t = 9.52$ that matches our derived global maximum condition of (17).

IV. OPTIMIZED SPACING IN SCMR BASED ON HELICES

In this section, we will examine the guidelines for designing helical TX and RX elements of SCMR wireless powering systems. An SCMR system based on helices will not be optimal unless the spacing s is picked so that the helices exhibit the appropriate capacitance in order to resonate at the desired operating frequency of the system. In [18], it was also shown that the spacing s of an SCMR helix is an important parameter that should be carefully picked to ensure optimal wireless power transfer efficiency. The capacitance formula for closely wound helix was presented in [19] as follows:

$$C_{\mathrm{t}} = \frac{2\pi^2 r \varepsilon_0}{\ln\left[\frac{s}{2r_{\mathrm{c}}} + \sqrt{\left(\frac{s}{2r_{\mathrm{c}}}\right)^2 - 1}\right]} \tag{21}$$

where r is the radius of the helix, r_{c} is the cross-sectional wire radius, ε_0 is the permittivity of free space, s is the spacing between adjacent turns of the helix, C_{t} is the total distributed capacitance of the helix, and t is the thickness of the insulation coating.

The capacitance formula of (21) is valid when $s/2r_{\mathrm{c}} \le 2$ and $t \ll s - 2r_{\mathrm{c}}$. In order to resonate the helix at a desired frequency f, the spacing between two adjacent turns, s, can be adjusted to provide the required capacitance calculated from (1) as

$$C_{\mathrm{t}} = \frac{1}{4\pi^2 f^2 L_{\mathrm{helix}}}. \tag{22}$$

Then, (21) can be solved for the spacing s as follows:

$$s = \frac{\left[e^{\left(4\pi^4 r^2 \varepsilon_0^2 / C_{\mathrm{t}}^2\right)} + 1 \right] r_{\mathrm{c}}}{e^{\left(2\pi^2 r \varepsilon_0 / C_{\mathrm{t}}\right)}}. \tag{23}$$

Equation (23) is valid when $s/2r_c \leq 2$ and $t \ll s - 2r_c$. Therefore, the spacing s can be adjusted using (23) independently from the other geometrical parameters to achieve the necessary capacitance and without affecting the frequency where a short helix or solenoid $(2r > h)$ exhibits maximum Q-factor since (13) shows that the $f_{\max}$ does not depend on s.

V. CONCLUSION

This letter analytically examines the optimal design of SCMR elements. Specifically, helices are analyzed. We analytically derived equations that can be used to design SCMR systems. In addition, our formulations proved that the global maximum Q-factor of a helix is achieved when the ratio between the radius r and the cross-sectional radius r_c of the helix is approximately equal to 9.52. This important finding can be used to design maximally efficient SCMR wireless powering systems that use coils or helices as transmitting and receiving elements.

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