Intracell Modeling of Lumped Elements Using the Composite-Cell MRTD Technique

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***Abstract* — In this paper a method is presented that allows the modeling of subcell lumped elements in Haar based multiresolution time-domain (MRTD). The method is applied through the use of a reconstruction/discrete wavelet transform algorithm that is integrated into the MRTD update scheme. A derivation of the MRTD method is presented for generic wavelet bases to demonstrate the application of the technique and suggest methods for using other wavelet bases. The Haar subcell technique is further presented as a bridge between point based updates (FDTD) and wavelet based multiresolution schemes (MRTD). Extensions of this technique include other point based algorithm modifications such as subcell PML and wire modeling.**

***Index Terms* — MRTD, FDTD, multiresolution, wavelet, subcell, composite cell**

I. INTRODUCTION

The multiresolution time-domain (MRTD) method [1] is a wavelet based technique that has the chief advantages of an inherent time- and space- adaptive grid and lower dispersion than methods of similar complexity. MRTD schemes have been developed for several wavelet bases, and each method has its own unique strengths. One of the main tradeoffs of wavelet bases is between algorithmic complexity and efficiency. Wavelet schemes, such as Battle-Lemarie [1] have been shown to require very few coefficients per wavelength, at the cost of a large stencil size (the number of basis functions that overlap). At the opposite end of this spectrum is Haar [2] wavelets, which have no overlap, but which also has dispersion properties equivalent to FDTD [3]. Another wavelet scheme which has been shown to have improved dispersion performance compared to FDTD, with a smaller stencil than Battle-Lemarie, is biorthogonal wavelets [4].

Regardless of the scheme used, one of the hallmark features of the MRTD scheme is a "cell size" that is larger than that of FDTD. In this document, the cell size is referred to as the domain of the scaling function. MRTD is an explicit time-domain technique, and as such boundary conditions of usually applied directly after field updates. A perfect electrical conductor (PEC) or perfect magnetic conductor (PMC) boundary condition, for example can be applied by setting the fields tangential to their locations to zero. As PEC features are usually significantly smaller than the maximum grid size, there is a disparity between the advantages of the method and the reality of its application.

One of the difficulties of applying the MRTD method is simulating fine features. Complex dielectric structures can be modeled through the discretization of the relationship between the electric field and electric flux density [1]. This relationship cannot be exploited, however, to implement hard boundary conditions, such as PECs. A technique has been presented that allows hard boundary conditions to be applied for Haar methods through the use of a reconstruction/discrete wavelet transform algorithm [5]. In this paper it is shown that this method provides a bridge between a pointwise update scheme, such as in FDTD, and the wavelet update scheme of MRTD. It is demonstrated in terms of general wavelet expansions that this technique can be expressed as an explicit application of the boundary condition on the wavelet field expansion. As an example of how this technique can be applied, it is extended to treat lumped elements.

II. MRTD BACKGROUND

While MRTD is a general term referring to the application of wavelets to the discretization of Maxwell's equations in the time-domain, the procedure for deriving the field update scheme is independent of the wavelet basis chosen. As such, the general derivation of MRTD methods is presented here. This paper presents a specific method that can be applied using Haar wavelets, but this general discretization is necessary to illustrate both how the method can be applied and what is unique in Haar wavelet discretizations that allows a pointwise application of specialized elements.

To derive an MRTD update scheme, the fields must first be expanded in terms of wavelets and scaling functions. Like the FDTD method, the electric and magnetic fields are offset in space and time. As a fixed time step is used, the time offset is $\Delta t/2$, as in FDTD, and the space offset will be, for now, left as s. For example, the expansion of the E_x component, in two spatial dimensions (for TE_z mode simulations) in terms of general scaling functions, φ, and wavelets, ψ, is

$$E_x(x) = \sum_{n,i,j} h_{n(t)} \cdot \begin{bmatrix} {}_nE^{x,\phi\phi}_{i,j}\varphi_i(x)\varphi_j(y) + \\ \sum_{r=0}^{r_{\max}}\sum_{p=0}^{2^r-1} {}_nE^{x,\psi\phi}_{i,j,r,p}\psi^r_{i,p}(x)\varphi_j(y) + \\ \sum_{r=0}^{r_{\max}}\sum_{p=0}^{2^r-1} {}_nE^{x,\phi\psi}_{i,j,r,p}\varphi_i(x)\psi^r_{j,p}(y) + \\ \sum_{r=0}^{r_{\max}}\sum_{p=0}^{2^r-1}\sum_{s=0}^{r_{\max}}\sum_{q=0}^{2^s-1} {}_nE^{x,\psi\psi}_{i,j,r,p,s,q}\psi^r_{i,p}(x)\psi^s_{j,q}(y) \end{bmatrix}, \quad (1)$$

$\varphi_i(x) = \varphi\left(x/_{\Delta x} - i\right)$ and $\psi^r_{i,p}(x) = 2^{r/2}\psi_0\left(2^{r/2}\left(x/_{\Delta x} - i\right) - p\right)$. The maximum level of wavelet resolution is r_{max}. It is convienient to use a compact notation of the field components [2] in the form of

$${}_n\mathbf{E}^x_{i,j} = \begin{bmatrix} {}_nE^{x,\phi\phi}_{i,j} \\ {}_nE^{x,\psi\phi}_{i,j,0,0} \\ \vdots \\ {}_nE^{x,\psi\psi}_{i,j,r_{\max},2^{r_{\max}-1}r_{\max},2^{r_{\max}-1}} \end{bmatrix} \quad (2)$$

that permits the update schemes to be represented as matrix equations.

The next step in the derivation of the update scheme is to insert these expressions into Maxwell's curl equations,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} - \mathbf{M} \quad (3)$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \cdot \quad (4)$$

Then the method of moments is applied. The inner product with the time pulse $h_n(t)$ is taken so that only one time step of the space derivative terms, and two of the time derivative terms remain in the equation. Then, the inner product with all space scaling functions and wavelets are taken. The locations of the scaling and wavelet functions are chosen such that the field under the time derivative is localized. The result is an explicit update equation for each wavelet/scaling component in terms of the surrounding fields. The update equation for the E_x field, in a homogeneous media takes the form

$${}_{n+1}\mathbf{E}^x_{i,j} = {}_n\mathbf{E}^x_{i,j} + \frac{\Delta t}{\varepsilon \Delta y}\sum_{a=-m}^{m}\left(\mathbf{U}_{E_{x1}\,n+\frac{1}{2}}\mathbf{H}^z_{i,j}\right)_{i,j+a,k} \cdot \quad (5)$$

It is important to note that (5) is a matrix equation. In this equation, m is the stencil size, determined by the overlap of the scaling and wavelet functions with neighboring scaling/wavelet functions. Haar-MRTD updates depend only on the fields at the same location, and thus m=0. Battle-Lemarie wavelets, on the other hand, are entire domain, although they are usually truncated with a stencil size of around 9.

In (5), **E** and **H** are column vectors, with $2^{2(r_{\max}+1)}$ elements (the number of scaling and wavelet coefficients. **U** is a square matrix, $2^{2(r_{\max}+1)}$ x $2^{2(r_{\max}+1)}$ in size. The elements of **U** are the coefficients of the inner products calculated using the method of moments. The values in the U matrix are the results of the inner products calculated with the method of moments.

The time step used in MRTD is dependent on the wavelet basis used and as such will not be covered here. The discussion presented in this paper is dependent on the spatial discretization, and as such the time step is not directly relevant. The other important parameter that has not been presented is the spatial offset between the electric and magnetic field. It has been shown [2,6], that the offset $1/2^{r_{\max}+2}$ leads to the best dispersion relationship. In the Haar case, this yields an equivalent dispersion as the FDTD method.

Of course, the chief advantage of the MRTD method is the time- and space- adaptive grid. This can be applied to the method that has been presented by adding and removing wavelet coefficients. An optimal method for varying wavelet resolution has not been presented, although existing methods use both a relative and absolute threshold. By periodically checking whether wavelets are required, the resolution can be varied with time.

Due to the fact that the resolution varies with time it is difficult to specify the offset based on the maximum resolution. Instead, the offset must be set on the characteristics of the initial grid. Increasing the resolution at this point will allow fine features and complex fields to be modeled, but with more dispersion than a grid formulated with the offset given above.

III. MRTD SUBCELL MODELING

The update presented in (5) is for the case of a homogeneous medium. In the more general case, (5) is not a direct update of E fields from H fields, but rather D from H. Methods have been presented [1] that then allow E components to be determined from the D components. However, the method provides no means for the simulation of hard boundaries. These conditions must be explicitly placed on the grid. In [5] a method is presented that allows the modeling of PECs at the equivalent grid point level in Haar-MRTD.

In order to represent PEC elements in the MRTD simulator, the electric fields that are tangential to PEC boundaries must be zeroed. In [5] this is applied by first reconstructing the Haar-MRTD electric fields, utilizing. $2^{D(r_{\max}+1)}$ (where D is the dimensionality of the simulator) equivalent grid points (areas with constant field value) per cell. The concept of equivalent grid points was first presented in [6]. This is the same number of points as scaling/wavelet coefficients, and, due to the orthogonality of the basis functions, they are independent. If the scaling/wavelet coefficients are arranged as in (2), a reconstruction matrix can be used to convert the scaling/wavelet coefficients to field values. The inverse of that matrix converts field values at equivalent grid points to MRTD coefficients, effectively performing the DWT.

In the general MRTD scheme, reconstruction and DWT can be viewed as operators instead of matrices. In this case, a similar condition could be applied by first reconstructing the wavelet values, zeroing fields in the appropriate areas, and then performing a DWT to obtain the MRTD coefficients. Of course, because a finite number of wavelet coefficients are used, it may not be possible to represent a given structure accurately with this method.

The technique presented in [5] can be generalized to allow pointwise effects to be applied at the MRTD equivalent grid points. The Haar-MRTD update equation, (shown here for the E_x field),

$$ {}_{\mathbf{n}}\mathbf{E}^{\mathbf{x}}_{\mathbf{i,j}} = {}_{\mathbf{n-1}}\mathbf{E}^{\mathbf{x}}_{\mathbf{i,j}} + \frac{\Delta t}{\varepsilon \Delta y}\mathbf{U}_{\mathbf{E}_{x1}\ \mathbf{n-1}}\mathbf{H}^{\mathbf{z}}_{\mathbf{i,j}} + \frac{\Delta t}{\varepsilon \Delta y}\mathbf{U}_{\mathbf{E}_{x2}\ \mathbf{n-1}}\mathbf{H}^{\mathbf{z}}_{\mathbf{i,j-1}} \tag{6} $$

can be converted to a pointwise form by multiplying with the reconstruction matrix, **R**,

$$ \begin{aligned} \mathbf{R}_{\mathbf{n}}\mathbf{E}^{\mathbf{x}}_{\mathbf{i,j}} &= \mathbf{R}_{\mathbf{n-1}}\mathbf{E}^{\mathbf{x}}_{\mathbf{i,j}} + \\ &\frac{\Delta t}{\varepsilon \Delta y}\left(\mathbf{U}'_{\mathbf{E}_{x1}\ \mathbf{n-1}}\mathbf{R}_{\mathbf{n-1}}\mathbf{H}^{\mathbf{z}}_{\mathbf{i,j}} + \mathbf{U}'_{\mathbf{E}_{x2}\ \mathbf{n-1}}\mathbf{R}_{\mathbf{n-1}}\mathbf{H}^{\mathbf{z}}_{\mathbf{i,j-1}}\right) \end{aligned} \tag{7} $$

if,

$$ \mathbf{U}' = \mathbf{R}\mathbf{U}\mathbf{R}^{-1} \tag{8} $$

At this point, the values represented by $\mathbf{R}_{\mathbf{n}}\mathbf{E}^{\mathbf{x}}_{\mathbf{i,j}}$ and $\mathbf{R}_{\mathbf{n-1}}\mathbf{H}^{\mathbf{z}}_{\mathbf{i,j}}$ represent the fields at equivalent grid points. An effect that affects a single or a small number of subcell grid points can be applied at this point. To multiply individual grid points by a discrete value, each field vector can be multiplied by a pointwise update matrix, **G**, to yield

$$ \begin{aligned} \mathbf{R}_{\mathbf{n}}\mathbf{E}^{\mathbf{x}}_{\mathbf{i,j}} &= \mathbf{G}_{\mathbf{E}_x}\mathbf{R}_{\mathbf{n-1}}\mathbf{E}^{\mathbf{x}}_{\mathbf{i,j}} \\ &+ \frac{\Delta t}{\varepsilon \Delta y}\begin{pmatrix} \mathbf{G}_{\mathbf{E}_x,\mathbf{H}_{z1}}\mathbf{U}'_{\mathbf{E}_{x1}\ \mathbf{n-1}}\mathbf{R}_{\mathbf{n-1}}\mathbf{H}^{\mathbf{z}}_{\mathbf{i,j}} \\ +\mathbf{G}_{\mathbf{E}_x,\mathbf{H}_{z2}}\mathbf{U}'_{\mathbf{E}_{x2}\ \mathbf{n-1}}\mathbf{R}_{\mathbf{n-1}}\mathbf{H}^{\mathbf{z}}_{\mathbf{i,j-1}} \end{pmatrix} \end{aligned}. \tag{9} $$

In the case with no pointwise effects, **G** is simply the identity matrix. In any scheme, only the diagonal elements of G are nonzero. To apply a PEC, the diagonal element in the row corresponding to the zeroed grid point is set to zero. The scheme can then be converted back to MRTD updates through multiplication with $\mathbf{R}^{-1}$, giving

$$ \begin{aligned} \mathbf{E}^{\mathbf{x}}_{\mathbf{i,j}} &= \mathbf{R}^{-1}\mathbf{G}_{\mathbf{E}_x}\mathbf{R}_{\mathbf{n-1}}\mathbf{E}^{\mathbf{x}}_{\mathbf{i,j}} \\ &+ \frac{\Delta t}{\varepsilon \Delta y}\begin{pmatrix} \mathbf{R}^{-1}\mathbf{G}_{\mathbf{E}_x,\mathbf{H}_{z1}}\mathbf{U}'_{\mathbf{E}_{x1}\ \mathbf{n-1}}\mathbf{R}_{\mathbf{n-1}}\mathbf{H}^{\mathbf{z}}_{\mathbf{i,j}} \\ +\mathbf{R}^{-1}\mathbf{G}_{\mathbf{E}_x,\mathbf{H}_{z2}}\mathbf{U}'_{\mathbf{E}_{x2}\ \mathbf{n-1}}\mathbf{R}_{\mathbf{n-1}}\mathbf{H}^{\mathbf{z}}_{\mathbf{i,j-1}} \end{pmatrix} \end{aligned}. \tag{10} $$

The matrices in (10) can be multiplied together,

$$ \mathbf{U}^{\mathbf{G}} = \mathbf{R}^{-1}\mathbf{G}\mathbf{U}'\mathbf{R} \tag{11} $$

and an update equation resembling (6) results

$$ \begin{aligned} {}_{\mathbf{n}}\mathbf{E}^{\mathbf{x}}_{\mathbf{i,j}} &= \mathbf{G}^{\mathbf{R}}_{\mathbf{E}_x\ \mathbf{n-1}}\mathbf{E}^{\mathbf{x}}_{\mathbf{i,j}} \\ &+ \frac{\Delta t}{\varepsilon \Delta y}\left(\mathbf{U}^{\mathbf{G}}_{\mathbf{E}_{x1}\ \mathbf{n-1}}\mathbf{H}^{\mathbf{z}}_{\mathbf{i,j}} + \mathbf{U}^{\mathbf{G}}_{\mathbf{E}_{x2}\ \mathbf{n-1}}\mathbf{H}^{\mathbf{z}}_{\mathbf{i,j-1}}\right) \end{aligned}. \tag{12} $$

A. Lumped Element Modeling

Using the procedure defined above, a number of pointwise effects can be applied to individual equivalent grid points in the MRTD method. For example, an equation for modeling z-directed resistors in a 3D FDTD scheme is [7]

$$ {}_{n+1}E^z_{i,j,k} = \left(\frac{1-\dfrac{\Delta t\Delta z}{2R\varepsilon_0\Delta x\Delta y}}{1+\dfrac{\Delta t\Delta z}{2R\varepsilon_0\Delta x\Delta y}}\right){}_nE^z_{i,j,k} + \left(\frac{\dfrac{\Delta t}{\varepsilon_0}}{1+\dfrac{\Delta t\Delta z}{2R\varepsilon_0\Delta x\Delta y}}\right)\nabla\times {}_{n+\frac{1}{2}}H_{i,j,k} \text{,} \tag{13} $$

This expression can be easily modified for x and y oriented resistors. A resistor can be applied at an individual grid point in the MRTD scheme by setting the appropriate row of the **G** matrix to the multiplier in (13). Similar expressions for inductors, capacitors, and diodes [7] can be applied as well.

Similar features that can be modeled with this method are thin wires [8], narrow slots [9], and many other techniques. Another important use of this method is that the perfectly matched layer [11] can be implemented such that that the material properties vary by equivalent grid point, and even fill only part of a cell. Thus, if a cell with 16 equivalent grid points per direction were used, it could have only 10 cells of PML. Most techniques that can be applied to the FDTD method can be extended to MRTD in this manner.

IV. EXAMPLE

A brief example is presented to validate the technique. In this example, an air-filled parallel plate waveguide (separation 30mm) is simulated in 2D (TE_z mode). The waveguide is terminated with a 50Ω resistor. The resistor does not match the waveguide, and thus reflection occurs. The time domain-results for MRTD r_{max}=2 and FDTD are presented in Fig. 1 and they are identical. The parallel plate waveguide is only one half of the MRTD cell in width (equivalent to 4 FDTD cells), and the resistors span 4 equivalent grid points.

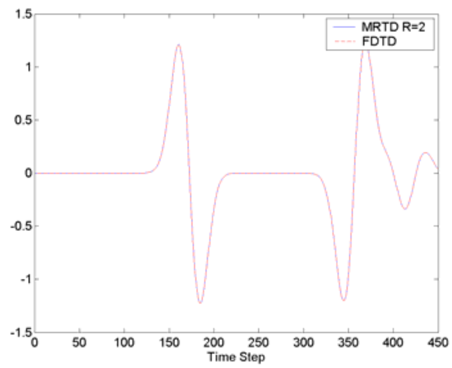

Fig. 1. Comparison of MRTD and FDTD simulation of parallel plate waveguide

In the final version of this paper, results from the simulations of periodically loaded transmission line with effective negative refractive index and negative group velocity a will be presented. Currently, there is much research interest in loaded transmission lines for meta-material based applications [11]. The fine details and large number of lumped components required for the simulation of these devices makes their modeling difficult and time-consuming. Using the method presented in this paper, a structure consisting of repeated elements of the unit-cell topology in Fig. 2. can be efficiently modeled. The variable grid and fine subcell features of the method are well suited to this problem and could lead to a much faster modeling and understanding of the physics of these structures..

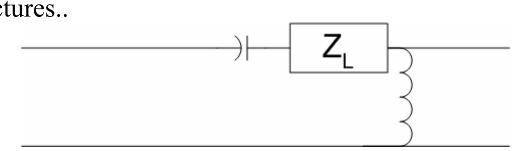

Fig. 2. Unit cell of a loaded microstrip line for meta-material applications

V. Conclusion

In this paper a method is presented that allows effects at the subcell level to be applied in Haar-MRTD. It is shown, through a general derivation of the MRTD method for any wavelet basis, that the technique comes from the use of reconstruction/decomposition operators. In the case of a general wavelet scheme it is necessary to determine what features can be accurately represented at the chosen resolution level when applying the technique. In the special case of Haar-MRTD, the algorithm effectively combines the pointwise nature of FDTD with the wavelet multiresolution nature of MRTD. This allows MRTD to use all of the extensions that have been for developed for MRTD while still taking advantage of the time- and space-adaptive resolution. Some of the techniques that can be modeled using this method are lumped elements, equivalent circuits, thin wires, narrow gaps, and the perfectly matched layer and can be used for the optimization of novel structures, such as metamaterials.

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References

[1] M. Krumpholz, L. P. B. Katehi, “New time domain schemes based on multiresolution analysis”, *IEEE Trans. Microwave Theory Tech.*, vol.44, pp.555-561, Apr. 1996

[2] T. Dogaru and L. Carin, “Application of Haar-wavelet-based multiresolution time-domain schemes to electromagnetic scattering problems,” *IEEE Trans. Antennas Prop.*, vol. 50, no. 6, pp.774-784, June 2002.

[3] K. S. Yee, “Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media,” *IEEE Trans. Antennas Prop.*, vol.AP-14, pp.302-307, Mar. 1966.

[4] T. Dogaru and L. Carin, “Scattering analysis by the multiresolution time-domain method using compactly supported wavelet systems,” *IEEE Trans. Microwave Theory Tech*, vol. 50, no. 7, pp.1752-1760 July 2002.

[5] N. Bushyager and M. Tentzeris, “Composite cell MRTD for the efficient simulation of complex microwave structures,” *Proc. 2003 European Microwave Conference,* pp. Oct. 2003.

[6] C. Sarris and L. P. B. Katehi, “Fundamental gridding-related dispersion effects in multiresolution time-domain schemes,” *IEEE Trans. Microwave Theory Tech*, vol. 49, no. 12, pp 2248-2257, Dec. 2001.

[7] M. Piket-May, B. Houshmand, and T. Itoh, “High-speed electronic circuits with active and nonlinear components,” ch. 15 in *Computational Electrodynamics: The Finite Difference Time Domain Method*, A. Taflove, Ed. Norwood, MA: Artech House, 1995.

[8] K. R. Umashankar, A. Taflove, and B. Beker, “Calcluation and experimental validation of induced currents on coupled wires in an arbitrary shaped cavity,” *IEEE Trans. Antennas and Prop.*, vol. 35, no. 11, pp. 1248-1257, Nov. 1987.

[9] A. Taflove, K. R. Umashankar, B. Bker et. al., “Detailed FDTD analysis of electromagnetic fields penetrating narrow slots and lapped joints in thick conducting screens,” *IEEE Trans. Antennas and Prop.*, vol. 36, pp. 247-257, Feb. 1998.

[10] J. P. Berenger, “A perfectly matched layer for the absorption of electromagnetic waves,” *J. Comp. Physics*, vol. 114, pp. 185-200, 1994.

[11] O. F. Siddiqui, M. Mojahedi, and G.V. Eleftheriades, “Periodically loaded transmission line with effective negative refractive index and negative group velocity,” *IEEE Trans. Antennas and Prop.*, vol. 51, no. 10, pp. 2619-2625, Oct. 2003.