

## Application of Bridge Function Sequences in Passive Beamformer

Sheng Hong\*, Huagang Xiong, Kefei Liu,  
Qing Chang, Qishan Zhang, Yongqiang Liu,  
School of Electronic and Information Engineering  
Beihang University  
Beijing, China  
\*fengqiao1981@gmail.com

Xiaoxiang He  
CIST, Nanjing Univer. of Aeronautics and Astronautics  
\*\*Manos M. Tentzeris  
GEDC/ECE, Georgia Institute of Technology,  
Atlanta, GA 30332-250, U.S.A.  
\*\*etentze@ece.gatech.edu

**Abstract**—This paper provides a new kind of passive beamformer which uses bridge function sequences as spreading sequence weights. The passive beamformer uses pseudorandom spreading sequence as weight of signal from each antenna element. Then it forms spatial radiation pattern by combining these signals. It can track and locate multiple objects' positions simultaneously without the need of phase shifters and attenuators and any adaptive electronic beamsteering. And its antenna array can be any arbitrary geometry displacement. It can discriminate directions of arrival with as high angle resolution as  $1.4^\circ$  when there are 2000 elements in the antenna array. We use bridge function sequences as its pseudorandom spreading sequence weights. Owing to Bridge function sequences with better cross correlation characteristics, the passive beamformer gets better performance promotion than ones with Walsh function sequences as spreading sequences.

**Keywords**- Passive beamformer, Bridge function sequences, Angle resolution, Synchronization error

### I. INTRODUCTION

Communication technologies are a swiftly developing field. The next generation communication system will be focused on the combination of smart antenna and OFDM technologies. Smart antenna can provide many good merits such as improved system capacities, space division multiple access (SDMA), higher signal-to-interference ratios, sidelobe canceling or null steering, instantaneous tracking of moving sources, improved array resolution, reduced speckle in radar imaging, clutter suppression. Traditional smart antennas are implemented by analog manner. The second generation smart antenna system carries out beamsteering with digital methods. This method is called as digital beamforming. The digital beamforming has overwhelming advantages over analog beamforming such that it saves many attenuators and phase shifters. But it has deficiencies like complicated and time-consuming adaptive digital beamforming algorithm. These algorithms are of no effect in case it can not track the users' motion fast enough. Aiming at the defects of previous generation smart antennas, Frank B. Gross and Carl M. Elam[1] propose a new passive beamformer adapting spreading sequences as weights. It can process multiple angles of arrival simultaneously without adaptive algorithm, and it is not limited by acquisition or tracking speeds. It also doesn't need analog attenuators and phase shifters. Any

arbitrary and/or random antenna array geometry can be incorporated into this new approach. In Frank's paper, he doesn't account for specific regulations for choosing spreading sequence as weights of this new beamformer. Nor does he tell the beamformers' reliability and robustness of different spreading sequences. However, the statistical and orthogonal properties of spreading sequences will aid in the identification of the exact directions and phase information of arrival signals[2]. Bridge function sequences are three valued spreading sequences taking the values -1, 0, and +1 which have correlation function with zero correlation zones so that they have better statistical performance. In our paper we make some research by adapting Bridge function sequences as array weights and investigating the performance difference between the beamformers respectively with Bridge function sequences and Walsh function sequences as weights.

The remainder is provided as following parts. II shows the principle for passive beamformer. III demonstrates Bridge function sequences. IV shows beamformer with Bridge function sequences. V is the conclusion.

### II. PRINCIPLE FOR PASSIVE BEAMFORMER

#### A. Overall Structure of Passive Beamformer

The passive beamformer[1] is described as Fig.1, wherein the array weights  $\beta_n(t)$  are orthogonal spreading sequences. It can be used with any arbitrary N-element antenna array. For purposes of illustration, the array used in this discussion will be an N-element linear array.

The incoming signals arrive at angles  $\theta_l$  where  $l = 1, 2, \dots, L$ . Each different angle of arrival produces a unique array element output with a unique phase relationship between each element. These phase relationships will be used in conjunction with the modulations  $\beta_n(t)$  to produce a unique summed signal  $y^r(t)$ .

#### B. Array Correlator Output

Each baseband output,  $X_n(t)$ , of the receive array will have a complex voltage waveform whose phase will consist of each emitter's phase modulation  $m_l(t)$  and the unique receive antenna element phase contributions. Ignoring the space loss and polarization mismatches, the received

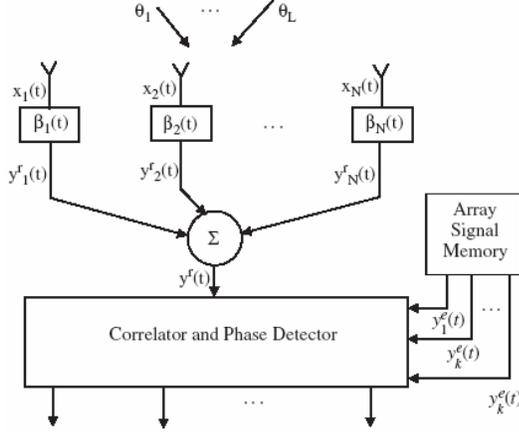


Fig. 1 Passive beamformer with spreading sequences as array weights.

baseband array output is given in vector form as

$$\bar{\mathbf{x}}^r(t) = \begin{bmatrix} 1 & \cdots & 1 \\ e^{jkd \sin(\theta_1)} & \cdots & e^{jkd \sin(\theta_L)} \\ \vdots & \ddots & \vdots \\ e^{j(N-1)kd \sin(\theta_1)} & \cdots & e^{j(N-1)kd \sin(\theta_L)} \end{bmatrix} \begin{bmatrix} e^{jm_1(t)} \\ \vdots \\ e^{jm_L(t)} \end{bmatrix} \quad (1)$$

$$= \bar{\mathbf{A}}^r \cdot \bar{\mathbf{s}}^r(t)$$

where  $m_l(t)$  is the  $l$ -th emitter's phase modulation,  $\theta_l$  the angle of arrival of the  $l$ th incoming signal,  $d$  the array element spacing,  $k$  the  $2\pi/\lambda$ ,  $\lambda$  the wavelength, the steering vector for direction  $\theta_l$  is  $\bar{\mathbf{a}}_l^r = [1, e^{jkd \sin(\theta_l)}, \dots, e^{j(N-1)kd \sin(\theta_l)}]^T$ ,  $\bar{\mathbf{A}}^r$  the matrix of steering vectors for all angles of arrival  $\theta_l$ .  $\bar{\mathbf{s}}^r$  the vector of arriving signal baseband phasors.

The spreading waveforms can be viewed as phase-only array weights. These array weights can be depicted as the vector  $\bar{\beta}(t)^T = [\beta_1(t) \dots \beta_N(t)]$ . The total spread array output is called the received signal vector and is given by

$$\mathbf{y}^r(t) = \bar{\beta}(t)^T \cdot \bar{\mathbf{x}}^r(t) \quad (2)$$

$$\bar{\mathbf{A}}^e = \begin{bmatrix} 1 & \cdots & 1 \\ e^{jkd \sin(\theta_1)} & \cdots & e^{jkd \sin(\theta_K)} \\ \vdots & \cdots & \vdots \\ e^{j(n-1)kd \sin(\theta_1)} & \cdots & e^{j(n-1)kd \sin(\theta_K)} \end{bmatrix}$$

$$= [\bar{\mathbf{a}}_1^e \quad \cdots \quad \bar{\mathbf{a}}_K^e] \quad (3)$$

Corresponding to the actual  $N$ -element antenna array is a second virtual array modeled in memory. The memory array is based on the calibrated array and has  $N$  virtual outputs for each expected direction  $\theta_k$  ( $k = 1, 2, \dots, K$ ). The expected directions are merely defined as directions of possible interest to the passive beamformer. These directions could be equally spaced in order to search over a range of angles or these expected directions could be the

direction of known sources of interest.

The array signal memory steering vectors are created based upon  $K$  expected angles-of-arrival  $\theta_k$ . In Eq.(3),  $\bar{\mathbf{A}}^e$  is the matrix of steering vectors for expected direction  $\theta_k$  and  $\bar{\mathbf{a}}_k^e$  the steering vector for expected direction  $\theta_k$ .

The memory has  $K$  complex output waveforms, one for each expected direction  $\theta_k$ . Each memory output is similar to the actual array output in Eq.(4) and is given by

$$\mathbf{y}_k^e(t) = \bar{\beta}(t)^T \cdot \bar{\mathbf{a}}_k^e(t) \quad (4)$$

The best correlations occur when the actual angle of arrival matches the expected angle of arrival. The correlation can be used as a discriminate for detection. Since the arriving signals have a random arrival phase delay, a quadrature correlation receiver should be employed such that the carrier phase is a non-issue[3]. The general complex correlation output, for the  $k$ th expected direction, is given as

$$R_k = \int_t^{t+T} \mathbf{y}^r(t) \cdot \mathbf{y}_k^{e*}(t) dt = |R_k| e^{j\phi_k} \quad (5)$$

where  $R_k$  is the complex correlation at expected angle  $\theta_k$  and  $\phi_k$  the average correlation phase at expected angle  $\theta_k$ .

The new SDMA passive beamformer does not process the incoming signals with phase shifters or beam steering. The correlation magnitude  $|R_k|$  is used as the discriminate to determine if a signal is present at the expected angle  $\theta_k$ . If the discriminate exceeds a predetermined threshold then a signal is deemed present and the phase is calculated. Since it is assumed that the emitter phase modulation (PM) is nearly constant over the code length ( $M\tau_c$ ), the correlator output phase angle is approximately the average of the emitter PM. Thus,

$$\phi_k = \arg(R_k) \approx \tilde{m}_k \quad (6)$$

where  $\tilde{m}_k = 1/T \int_t^{t+T} m_k(t) dt$ , the average of emitter's modulation at angle  $\theta_k$ .

We take a signal transmitting at  $0^\circ$  direction towards an antenna array with 2000 elements for example and get the array correlation output. The angular resolution is as accurate as  $1.4^\circ$  so that it can discriminate 128 input directions meantime when the observation range covers from  $-90^\circ$  to  $90^\circ$ .

### III. BRIDGE FUNCTION SEQUENCES

#### A. Overview of Bridge Function Sequences

The Bridge function sequences are three valued functions taking the values -1, 0, and +1. These functions were introduced by Zhihua and Qishan[4] and are derived from a combination of the block pulses and the Walsh-Hadamard functions.

Just like Walsh-Hadamard functions, the Bridge function sequences are orthogonal functions and can be represented by an  $N \times N$  square matrix as follows[5].

$$B_N(j) = \begin{bmatrix} b_{0,0} & b_{0,1} & \cdots & b_{0,N-1} \\ b_{1,0} & b_{1,1} & \cdots & b_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ b_{N-1,0} & b_{N-1,1} & \cdots & b_{N-1,N-1} \end{bmatrix} \quad (7)$$

In terms of the generating method and sequence structure, bridge function sequences and Walsh function sequences have many similarities. Based on different copy and shift method there are four different kind of bridge function sequences with different numbering, among which the  $H\#$  bridge function sequences can be obtained with the following recursive procedure,

$$\begin{cases} \mathbf{H}_P^B(j) = \mathbf{I}_P, 0 \leq j \leq q \\ \mathbf{H}_{2^i P}^B(j) = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{H}_{2^{i-1} P}^B(j) & \mathbf{H}_{2^{i-1} P}^B(j) \\ \mathbf{H}_{2^{i-1} P}^B(j) & -\mathbf{H}_{2^{i-1} P}^B(j) \end{bmatrix} \\ i = 1, 2, \dots, q-j \end{cases} \quad (8)$$

where  $P=2^j$ ,  $\mathbf{I}_P$  is  $P$ -order unit matrix,  $j$  is shift parameter,  $N=2^q$  is the length of bridge function sequences generated.

$H_{2^q}^B(j)$  is called bridge function sequences matrix, each of whose column vectors corresponds with a Bridge function sequences. All of the  $2^q$  sequences in this matrix form a sequence set. Different shift parameter generates different bridge function sequences set, so  $q+1$  sequence set are generated totally. Specially, Walsh function sequence set is generated at  $j=0$ , while block pulse sequence is generated at  $j=q$ . Different Bridge function sequences is orthogonal to each other, and zero correlation zone (ZCZ) exists. Besides, Bridge function sequences contains zero value, and nonzero values ( $\pm 1$ ) are uniformly-spaced with  $2^j - 1$  zeros between adjacent elements.

### B. Correlation Function Comparison between Walsh function sequences and Bridge function sequences

Walsh function sequences are popularly used spreading

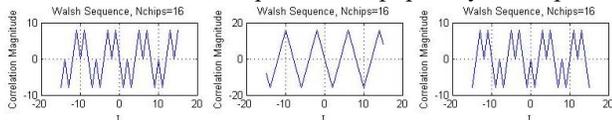


Fig. 2. Cross-correlation function of Walsh function sequences with 16 chips.

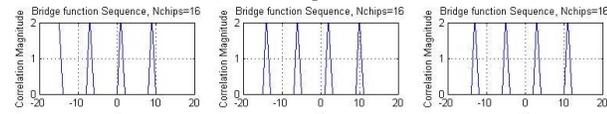


Fig. 3. Cross-correlation function of Bridge function sequences with 16 chips.

sequence in many fields. Owing to its good cross-correlation, it is used as spreading spectrum sequence in 2G and 3G communication systems. It is also used as orthogonal channelization sequence in CDMA2000 standard of 3G[6].

We compare the correlation function between Walsh function sequences and Walsh function sequences as follows. Apparently seen in Fig. 2 and Fig. 3, there are some zero correlation zones among correlation peaks in the cross-correlation function diagram of Bridge function sequences. Zero cross-correlation zone means less influence of synchronization error and multiple access interference.

## IV. PASSIVE BEAMFORMER WITH BRIDGE FUNCTION SEQUENCES

### A. Multiple Directions Detection and Waveform Estimation

Firstly, we take an uplink with single input direction at  $0^\circ$  into account. Fig. 4 illustrates the correlator's output of a correlation peak.

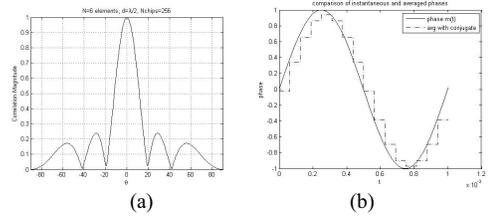


Fig. 4. Correlation detection and waveform estimation for single input direction at  $0^\circ$ : (a) Modulus of correlator output, (b) Phase information of correlator output.

We investigate the performance between Walsh function sequences and Bridge function sequences as follows. We respectively put both sequences into the passive beamformer and compare the synchronization error influence to these two systems. The simulation conditions are: single signal  $\sin(x)$  at  $0^\circ$  is transported without noise. As shown in Fig. 4, we could clearly discriminate the sharp correlation peak is with low level sidelobe and the samples of phase information of correlation output forms the original sinusoidal envelope.

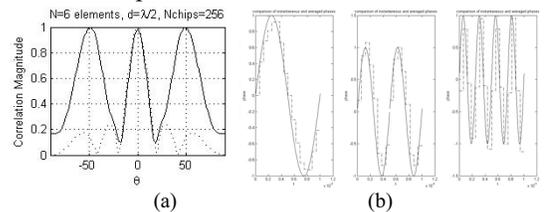


Fig. 5. Correlation detection and waveform estimation for multiple input directions at  $45^\circ, 0^\circ, -45^\circ$ : (a) Modulus of correlator output, (b) Phase information of correlator output.

We can also simulate the scenario of multiple inputting directions. We assume that there are 3 input sinusoidal signals at  $45^\circ, 0^\circ, -45^\circ$ . As shown in Fig. 5, the three input directions are detected by correlation output with 3 correlation peaks and the three input envelopes is approximated by samples of phase information of correlation outputs.

### B. Synchronization Error Influence

There is a very important issue that all the samples of

each channel should be synchronized in realistic experiments. Otherwise, the correlation detection performance will suffer a degradation. But it is hard to align all the clock signals at an identical timing trigger especially when the number of elements is large enough and the clock lines have to undertake a long way to arrive at different elements. There is probably a small synchronization defect existing. Hence, we investigate the influence of synchronization error and want to find a spreading sequences with better reliability and robustness. Fig. 6 and Fig. 7 show the influence of 2 clock defects respectively occurring in Walsh function sequences and Bridge function sequences when the input signal is coming at  $0^0$  direction. We definitely

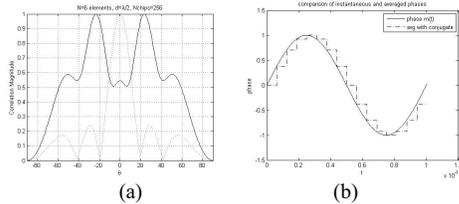


Fig. 6. Correlation detection and waveform estimation for single input directions at  $0^0$ : (a) Modulus of correlator output, (b)Phase information of correlator output.

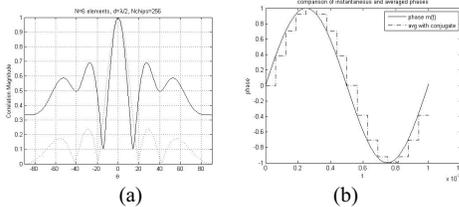


Fig. 7. Correlation detection and waveform estimation for single input directions at  $0^0$ : (a) Modulus of correlator output, (b)Phase information of Correlator output.

find that the passive beamformer using Walsh function sequences with 2 clock defects suffers worse degradation than the one using Bridge function sequences with 2 clock defects.

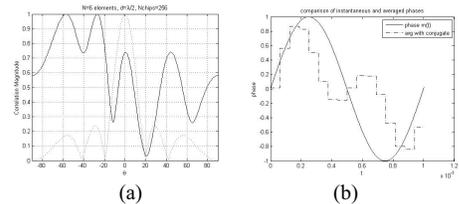


Fig. 8. Correlation detection and waveform estimation for multiple input directions at  $45^0$  and  $0^0$ : (a) Modulus of correlator output, (b)Phase information of correlator output at  $0^0$ .

We can also compare the same scenario when there are 2 input sinusoidal signals at directions of  $45^0$  and  $0^0$ . We can see from Fig. 8 that the passive beamformer using Walsh function sequences suffers a big interference so that we can not discriminate the input directions. As seen in Fig. 9, however, the passive beamformer using Bridge function sequences gets a better tolerance so that we can still detect the two input directions although it also suffers a

performance degradation.

We can see from the figures mentioned above that Bridge function sequences has better capability to be resistant to synchronization error. Its robustness comes from its characteristics that it has zero correlation zones among correlation peaks. It is very suitable for configuration as spreading sequence in this new passive beamformer.

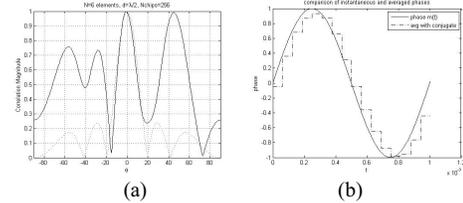


Fig. 9. Correlation detection and waveform estimation for multiple input directions at  $45^0$  and  $0^0$ : (a) Modulus of correlator output, (b)Phase information of correlator output at  $0^0$

## V. CONCLUSION

This paper provides a new passive beamformer using Bridge function sequences. Bridge function sequences have zero correlation zones among correlation peaks so that they have some anti-interference capability. We compare the performance of passive beamformer using Bridge spreading sequences with the one using Walsh spreading sequences. According to the simulation results, the proposed method has better capability of avoiding synchronization error. Setting Bridge function sequences as weights in the passive beamformer is suitable for usage in antenna array with a large number of elements. And it apparently boosts the reliability and robustness of system.

## ACKNOWLEDGMENT

This paper is supported by China National Science Fund with No.60872062 and 863 National High-tech Research and Development Program with No.2007AA12Z340.

## REFERENCES

- [1] Frank B. Gross, Carl M. Elam, "A new digital beamforming approach for SDMA using spreading sequence array weights," *Signal Processing*, v 88, n 10, Oct. 2008, pp. 2425-2430
- [2] Merrill Skolnik, *Radar Handbook, second ed.*, McGraw-Hill, New York, 1990, pp. 10.17 - 10.26
- [3] Haykin, Simon, *Communication Systems, second ed.*, Wiley, New York, 1983, pp.580.
- [4] Zhihua Li, Qishan Zhang, "Introduction to Bridge Functions", *IEEE Trans Electromagnetic Compatibility*, Vol EMC-25, Issue 4, Nov. 1983 Page(s):459 - 464
- [5] Slimane B S and Abdullatif G, "Multi-Carrier CDMA Systems Using Bridge Functions," *IEEE VTC Proceedings*, vol 3, pp. 1928-1932, 2000
- [6] Amadei, M., Manzoli, U., Merani, M.L., "On the assignment of Walsh and quasi-orthogonal codes in a multicarrier DS-CDMA system with multiple classes of users", *Global Telecommunications Conference, 2002*, vol 1, 17-21 Nov. 2002 pp:841 - 845