The Application of Lumped Element Equivalent Circuits Approach to the Design of Single-Port Microstrip Antennas

Gerald R. DeJean, *Member, IEEE*, and Manos M. Tentzeris, *Senior Member, IEEE*

***Abstract*—A simple method for inverting negative resistances of nonphysical circuits generated from approximating rational functions through vector fitting is proposed. This technique is applied to three resonant antenna structures. The challenge of applying this method to these structures lies in the accurate modeling of the ways in which the different resonant modes are excited, some via coaxially feeding a rectangular patch along the diagonal, while others via capacitively coupling energy from the driven patch to a parasitic patch. The equivalent circuits of these designs produce scattering parameter results that are consistent with the fitted functions. This methodology of resistance invertibility is a great tool that can be used to model antenna designs with low order equivalent circuits, drastically reducing the design time. In addition, valuable information that could aid in the optimization of microstrip antennas can be quickly ascertained through these techniques.**

***Index Terms*—Approximation order, equivalent circuit, microstrip antenna, rational approximations.**

I. Introduction

THE continuing growth in the design and use of microstrip antennas and resonator circuits for a wide range of shapes, integrated modules and applications has necessitated the need for increased research to fully interpret the physics behind the structures. The use of computational methods, such as the finite difference time domain (FDTD) [1], the method of moments (MoM) [2], the finite element method (FEM) [3], and the transmission line matrix (TLM) method [4], has aided in the understanding of microstrip antennas from the electromagnetics viewpoint. These computational methods have been integrated into commonly used computer-aided design (CAD) packages, such as High Frequency System Simulator (HFSS) [5], Sonnet Suite [6], and MicroStripes [7]. In order to take full advantage of analyzing resonant structures and patch antennas, it is necessary to couple these computational methods with circuit analysis techniques that could potentially provide researchers with additional information to explain phenomena (such as higher-order modes and parasitics) that may be present due to lossy substrates or complicated discontinuities or metal surfaces. In the past, Garg *et al.* presented design equations for analyzing microstrip antennas based on the parallel resistor-inductor-capacitor (RLC) circuit representation [8]. Additionally, Pozar has explained how microstrip circuits can be represented by open-circuited half wavelength $(\lambda/2)$ transmission lines [9]. The scattering (S-) parameter data from these circuit representations do not always agree with the data obtained from the computational full-wave electromagnetic methods. Hence, there is a need for improved circuit representations to bridge the gap between computational and circuit analysis, while accelerating the design process. Approximating rational functions through vector fitting is a great tool that can be used to create equivalent circuits based on scattering parameter data. A major limitation in the creation of these equivalent circuits is the presence of negative resistors which makes the circuit non-physical [10].

In this paper, a simple method is proposed that addresses the issue of unphysical negative resistances and can be applied to the design and analysis of single-port TM_{10} mode microstrip antennas and other resonant structures for the extraction of equivalent circuits that agree very well with full-wave EM simulation software. After a brief discussion of approximating admittance data with rational functions to generate a passive circuit is presented, the proposed method of resistance invertibility is discussed. From there, the proposed technique is applied to two simple resonant patch antenna designs. Lastly, some comments on the role of the order number and the number of necessary iterations are presented.

II. Approximating a Rational Admittance Function to Generate a Passive Circuit

The values of the lumped passive components for an equivalent circuit are determined through the analysis of approximating rational functions by vector fitting. A brief discussion on how this method works is posed in this section. A thorough investigation on approximating rational functions by vector fitting, enforcing passivity and generating a lumped equivalent circuit of passive elements is presented in [11] and [12].

The first step in this process is to acquire the scattering, impedance or admittance parameter data versus frequency of a design via full-wave electromagnetic simulation. There is no rule for the number of frequency points that should be sampled, but it is suggested to use no fewer than 100. Based on the lumped network circuit model used in this paper that is derived from a single-port admittance and to maintain consistency with previously published articles, the scattering or impedance parameter data has to be transformed to admittance parameter

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G. R. DeJean is with Microsoft Research, Redmond, WA 98052 USA (e-mail: dejean@microsoft.com).

M. M. Tentzeris is with the Georgia Electronic Design Center, School of Electrical and Computer Engineering, Georgia Institute of Technology, GA 30332 USA (e-mail: etentze@ece.gatech.edu).

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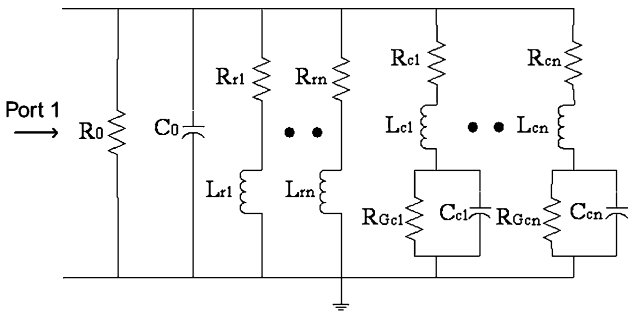

Fig. 1. Schematic of single-port equivalent circuit representation.

data. The importance of using this method for single-port antennas is noted because many antenna applications require single feed points. Then, a rational function approximation, $y_{\text{fit}}(s)$ is used to approximate the admittance parameter data. This function is shown below

$$y_{\text{fit}}(s) = \sum_{n=1}^{N} \frac{c_n}{s - a_n} + d + se \tag{1}$$

where s is a single frequency point $(= j\omega)$, c_n, and a_n are the residue and pole values, respectively from $n = 1, 2, \ldots, N$ where N is the number of poles (order of approximation), and d and e are higher order coefficients. The residue and pole values can either be real or exist in complex pairs (a complex number and its conjugate). In the beginning, (1) is nonlinear since the unknown poles, a_n, are in the denominator. To convert this function into a linear one that could be solved with straightforward techniques, the poles are transformed into known quantities by giving them values at the beginning of the fitting process. These are called starting poles $\overline{a}_n$, and they exist only at the start of the evaluation. These starting poles allow c_n, d, and e to be obtained by linearly solving the matrix equation, $Ax = b$, through singular value decomposition (SVD). In addition, a new set of poles, a_n, are created through an additional step described in [11]. After one iteration, the starting poles are replaced by this new set of poles and c_n, d, and e are solved again by SVD and the process continues in an iterative manner depending on the number of iterations desired in the fitting.

After the final c_n, a_n, d, and e parameters have been obtained, these values are correlated to an equivalent circuit shown in Fig. 1. Design equations for transforming these parameters to the lumped elements of Fig. 1 are presented in [12] and [13], while the concept of forming circuits from rational factors is shown in [14]. The $R_r - L_r$ branch corresponds to a real pole, while the $R_c - L_c - R_{Gc} - C_c$ branch represents a complex pole pair. Often times in this circuit, the R_c and R_{Gc} components are negative which makes the circuit nonphysical. Inverting these resistances to positive values can be used in some cases to create physical circuits without losing the profile of the admittance (and hence, the scattering parameter) data. This method is discussed in the next section.

III. Resistance Invertibility

In order to transform equivalent circuits with negative resistance values to physical circuits, a new method is proposed. This method involves inverting the negative resistances to positive values. The circuit element that is used to determine whether this method can be done is the value of R_c. If the magnitude of this value is less than around 100 Ω, then both R_c and R_{Gc} can be inverted to produce an accurate physical circuit. This is a safe approximation based on analysis of generating circuits with other values of R_c, in which, the circuit did not correlate well with the approximation. After analyzing the equivalent circuits of many antenna structures of various orders, it is observed that the value of R_{Gc} is important for obtaining an accurate circuit, but it is not significant in determining whether the inverting technique can be used. The reason why R_c and R_{Gc} are examined is because often times, the final poles used to generate the equivalent circuit fitted with an order between 2nd-Nth, often result in at least one complex pole pair which is represented by the $R_c - L_c - R_{Gc} - C_c$ branch.

In an attempt to develop a methodology for producing physical circuit from negative resistances, experiments were conducted of many antenna configurations where the approximation order and iteration number were varied. From that experimentation, the authors noticed a strong correlation between low order approximations, first resonant mode structures, and the value of R_c is observed. The significance of using an order number less than four to approximate simple antenna designs has been discussed, but it was observed that the magnitude of negative values of R_c must be small (less than 100 Ω) for the resistances to be inverted. Despite the fact that in some of the other designs, the magnitude of the negative R_{Gc} value is quite large, the investigation showed that as long as the values of R_c maintain a small value $(< 10^2\ \Omega)$ the equivalent circuit would still be accurate. In addition, extensive numerical experiments demonstrated that after negative R_c and R_{Gc} values have been inverted, one may still use positive resistance values within $\pm 5\%$ of the inverted resistances to produce an equivalent circuit with insignificant variation in the S-parameter results. The major limitation of this approach is the fact that this technique is not systematic. In other words, the method does not contain a physics-based approach, and therefore, it is expected that this method may not provide accurate results for all geometries. Nevertheless, it is stressed that this method can be used as a tool to aid engineers in designing an actual passive circuit that can be used to mimic the scattering parameter response of a patch antenna.

IV. Applications

The proposed method of inverting resistances is applied to the simulated designs of two microstrip patch antennas operating at around 2.4 GHz. The first structure is shown in Fig. 2. The $\lambda/2$ patch has a length L, of 950 mils and a width W, of 1050 mils. The substrate is a 76 mil thick layer of LTCC $(\varepsilon_r = 5.4, \tan\delta = 0.0015)$. The S-parameters were obtained through simulation using MicroStripes 7.0, a 3D full-wave simulator that uses the TLM method to solve for the electromagnetic fields. After transferring the S-parameters to admittance parameters, the data was fitted to a 4th order approximation using the following starting poles: $-2.2e7 \pm j * 2.2e9$ and $-2.6e7 \pm j * 2.6e9$. Lower orders of approximation of the admittance data were inaccurate and higher orders are unnecessary.

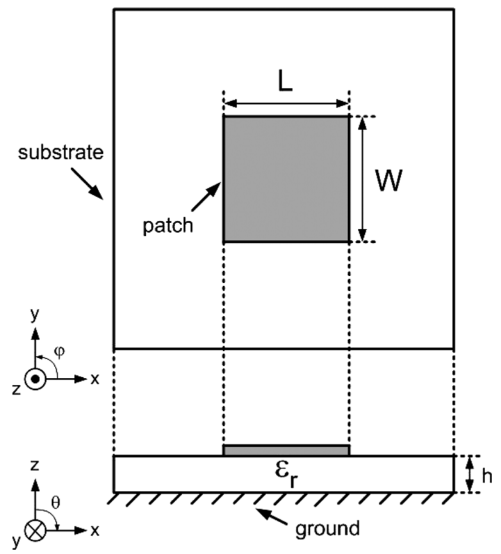

Fig. 2. Illustration of simple microstrip antenna.

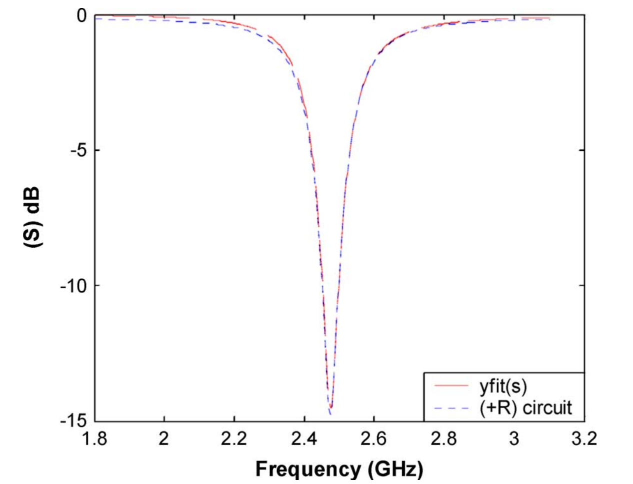

Fig. 3. S-parameter plot comparing the fitted function ($y_{\rm fit}(s)$) and the positive resistance equivalent circuit (1st row) and its magnitude deviation (2nd row) of patch antenna at around the 2.4 GHz band.

TABLE I

4^{th} order	**CPP Branch 1**	**CPP Branch 2**
R_c (Ω)	0.19	-6.11
L_c (H)	2.54e-9	3.04e-8
R_{Gc} (Ω)	-74.92	-32597
C_c (F)	2.43e-11	1.29e-13

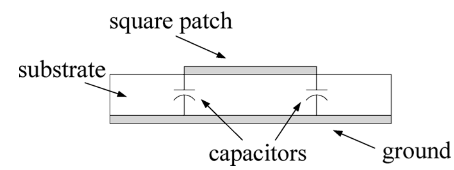

Fig. 4. Illustration of a microstrip antenna with tuning capacitors.

Three iterations were carried out in this fitting process. The 4th order approximation produced two complex pole pair branches as well as a resistor branch ($R_0 = 8.35$ kΩ) and a capacitor branch ($C_0 = 174.15$ fF). Table I show the passive component values of the complex pole pair branches.

In this table, the existence of negative resistor values is present. Consider the fact that the antenna being modeled has a dominant TM_{10} mode. Therefore, the technique of inverting the negative resistance values to positive resistance values is applied, hence, obtaining a physical equivalent circuit. Through experimentation, it has been observed that this technique works exceptionally well with conventional microstrip antennas with dominant TM_{10} resonant modes predicting accurately the resonant frequencies/bandwidths as well as the magnitude of the return loss. Agilent ADS2005 was used to simulate the equivalent circuit. A comparison between the S-parameters of the positive resistance circuit and $y_{\rm fit}(s)$ versus frequency is shown in Fig. 3. The positive resistant circuit agrees well with $y_{\rm fit}(s)$. The magnitude deviation is on the order of 10^{-2}.

Next, a microstrip antenna with tuning capacitors was simulated with a two resonant frequencies around 2.4 GHz. This schematic of this antenna is shown in Fig. 4. By feeding the antenna along the diagonal of the patch, two resonant modes are excited (TM_{10} and TM_{01}). The dimensions of the antenna are the same as that of the antenna in Fig. 2. The value of both capacitors is 280 fF. After the admittance data was acquired, an 8th order approximation was applied using three iterations. The starting poles for this fitting are $-2.2e7 \pm j * 2.2e9$, $-2.3e7 \pm j * 2.3e9$, $-2.4e7 \pm j * 2.4e9$, and $-2.5e7 \pm j * 2.5e9$. This approximation produced four complex pole pairs, a resistive branch ($R_0 = 180.81$ kΩ), and a capacitor branch ($C_0 = 114.11$ fF). Table II shows the component values of the complex pole pair branches.

The resistors in these branches are inverted, and the circuit is simulated. The return loss versus frequency for the inverted circuit and $y_{\rm fit}(s)$ is displayed in Fig. 5. There is a disagreement in the return loss around 2.32 GHz, but this discrepancy is on the order of 10^{-2}.

V. Significance of Approximation Order and Number of Iterations

When an equivalent circuit is created, the number of passive elements used is dependent on the order of the approximation; hence, it is necessary to discuss the significance of this order. Upon performing research on many antenna designs, including the ones presented in this paper, it is concluded that there is a tradeoff in the order number that is selected. If the order number is too small, it may be difficult to fit the peaks that exist in the admittance data. On the other hand, if the order is too large, more branches are created which can lead to more negative resistances that are too large to be inverted. Of course, the number of poles will depend on the order number (N poles $= N$th order). The significance on the number of iterations used in the fitting process is small if the magnitudes of the starting poles lie in the frequency range of the approximation. (If the approximation is between 1–2 GHz, for instance, then the magnitude of the starting poles must lie between 1–2 GHz.) The passive elements produced by the fitting tend to stabilize after the 2nd or 3rd iteration. The antenna in Fig. 2 was fit using one through five iterations. One circuit was simulated for each number of iterations. The

TABLE II

8^{th} order	**CPP Branch 1**	**CPP Branch 2**	**CPP Branch 3**	**CPP Branch 4**
R_c (Ω)	-0.03	-56.23	-5.98	481.35
L_c (H)	2.59e-9	1.13e-7	3.62e-8	2.00.e-7
R_{Gc} (Ω)	-322.92	1.15e6	-23905	-39783
C_c (F)	2.05e-11	4.07e-14	1.12e-13	1.01e-14

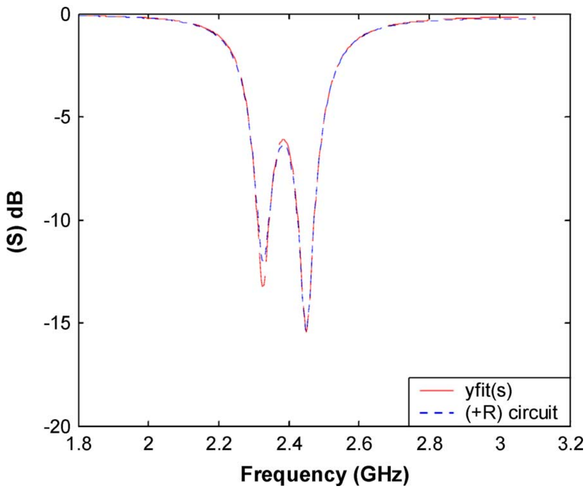

Fig. 5. S-parameter plot comparing $y_{\rm fit}(s)$ and the positive resistance equivalent circuit (1st row) and its magnitude deviation (2nd row) of the design in Fig. 4.

passive element values of the 3rd iteration are similar to those of the 4th and 5th iteration. Therefore, the circuits generated after the 2nd iteration had the best match to the approximated function with the smallest computational overhead.

VI. Conclusions

A simple method of inverting negative resistances in the equivalent circuit modeling of dominant TM_{10} mode microstrip antennas is proposed. After a specific topology of the studied design is simulated using computational electromagnetic solving techniques, the admittance data is approximated by a rational function through a process called vector fitting. The passive elements are generated from the rational function and applied to two patch antenna designs operating at around 2.4 GHz. When resistance invertibility is enforced, the equivalent circuit of this structure agrees well with the approximated function. Future work will focus on applying these methods to generate circuits to fit the parasitic effects of complex two-port antennas and high Q resonant structures with accurate models. These techniques have the potential to be used in the understanding of electromagnetic structures by analyzing the electrical performance of the equivalent circuit.

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Gerald R. DeJean (S'03–M'07) received the B.S. degree (*suma cum laude*) in electrical and computer engineering from Michigan State University, East Lansing, in 2000 and the M.S. and Ph.D. degrees in electrical and computer engineering from the Georgia Institute of Technology, Atlanta, in May 2005 and January of 2007, respectively.

He currently works at Microsoft Research as a Researcher in the field of RF design. His research interests include antenna design, RF/microwave design and characterization, and 3-D system-on-package (SOP) integration of embedded functions that focuses largely on modern commercial RF systems such as cellular phones for PCS applications, Bluetooth and 2.4 GHz ISM applications, RFID's, WLAN (802.11a,b,g), LMDS, and millimeter-wave applications at 60 GHz. He has dedicated his graduate research on making the antenna more compact and integrable with multilayer packages such as low temperature cofired ceramic (LTCC), liquid crystal polymer (LCP), and multilayer organic (MLO) while maintaining the full functionality of the device for wideband and/or multiband applications. He is also interested in equivalent circuit modeling techniques to assist in the design and optimization of compact antennas. He has authored and coauthored over 40 papers in refereed journals and conference proceedings. He has worked on a number of research projects as a member of the NSF Packaging Research Center and the Georgia Electronic Design Center.

Dr. DeJean is a member of the Eta Kappa Nu and Tau Beta Pi National honor societies. He was awarded the prestigious Microsoft Research Fellowship Award for excellence in graduate research and was twice a finalist in the student paper competition of the 2004 and 2005 IEEE Antennas and Propagation Society Symposia.

Manos M. Tentzeris (S'89–M'98–SM'03) received the diploma degree in electrical and computer engineering (*magna cum laude*) from the National Technical University of Athens, Greece, and the M.S. and Ph.D. degrees in electrical engineering and computer science from the University of Michigan, Ann Arbor.

He was a Visiting Professor with the Technical University of Munich, Germany for summer 2002, where he introduced a course in the area of high-frequency packaging. He is currently an Associate Professor with the School of Electrical and Computer Engineering, Georgia Institute of Technology (Georgia Tech), Atlanta. He is the Georgia Tech NSF-Packaging Research Center Associate Director for RF Research and the RF Alliance Leader. He is also the leader of the RFID Research Group of the Georgia Electronic Design Center (GEDC) of the State of Georgia. He has given more than 40 invited talks in the same area to various universities and companies in Europe, Asia and America. He has published more than 250 papers in refereed Journals and Conference Proceedings, one book and eight book chapters and he is in the process of writing two books.

Dr. Tentzeris is a member of the International Scientific Radio Union (URSI) Commission D, an Associate Member of EuMA, and a member of the Technical Chamber of Greece. He has helped develop academic programs in Highly Integrated/Multilayer Packaging for RF and Wireless Applications, Microwave MEM's, SOP-integrated antennas and Adaptive Numerical Electromagnetics (FDTD, MultiResolution Algorithms) and heads the ATHENA research group (15 researchers). He is the Georgia Electronic Design Center Associate Director for RFID/Sensors research, the Georgia Tech NSF-Packaging Research Center Associate Director for RF Research and the RF Alliance Leader. He is also the leader of the RFID Research Group of the Georgia Electronic Design Center (GEDC) of the State of Georgia. He was the recipient of the 2006 IEEE MTT Outstanding Young Engineer Award, the 2004 IEEE Transactions on Advanced Packaging Commendable Paper Award, the 2003 NASA Godfrey "Art" Anzic Collaborative Distinguished Publication Award for his activities in the area of finite-ground low-loss low-crosstalk coplanar waveguides, the 2003 IBC International Educator of the Year Award, the 2003 IEEE CPMT Outstanding Young Engineer Award for his work on 3D multilayer integrated RF modules, the 2002 International Conference on Microwave and Millimeter-Wave Technology Best Paper Award (Beijing, CHINA) for his work on Compact/SOP-integrated RF components for low-cost high-performance wireless front-ends, the 2002 Georgia Tech-ECE Outstanding Junior Faculty Award, the 2001 ACES Conference Best Paper Award and the 2000 NSF CAREER Award for his work on the development of MRTD technique that allows for the system-level simulation of RF integrated modules and the 1997 Best Paper Award of the International Hybrid Microelectronics and Packaging Society for the development of design rules for low-crosstalk finite-ground embedded transmission lines. He was also the 1999 Technical Program Co-Chair of the 54th ARFTG Conference, Atlanta, GA and the Chair of the 2005 IEEE CEM-TD Workshop and he is the Vice-Chair of the RF Technical Committee (TC16) of the IEEE CPMT Society. He has organized various sessions and workshops on RF/Wireless Packaging and Integration in IEEE ECTC, IMS and APS Symposia in all of which he is a member of the Technical Program Committee in the area of "Components and RF". He will be the TPC Chair for IEEE IMS 2008 Symposium. He is the Associate Editor of IEEE TRANSACTIONS ON ADVANCED PACKAGING. He has given more than 50 invited talks in the same area to various universities and companies in Europe, Asia and America.