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APPLICATION OF THE PML ABSORBER TO THE MRTD TECHNIQUE

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Abstract

The PML absorbing boundary condition is used in the two-dimensional MRTD technique in order to analyze open planar structures. The results are compared to those obtained by use of the conventional FDTD technique and substantial reductions in memory and execution time requirements are observed.

I Introduction

Recently, new multiresolution time domain (MRTD) schemes [1], based on orthonormal wavelet analysis have been applied to the analysis of various types of microwave structures. The inherent properties of these schemes provide advantages over Yee's finite-difference-time-domain (FDTD) technique with respect to memory requirements and execution time. [1]. The object of this paper is to demonstrate that these advantages exist in the case of open planar structures.

For the derivation of the MRTD scheme, the electromagnetic fields are represented by an expansion in cubic spline Battle-Lemarie scaling functions [2] with respect to space. Pulse functions are used as expansion and test functions in time-domain. For this type of scaling functions, the evaluation of the moment method integrals is simplified due to the existence of closed form expressions in spectral domain and simple representations in terms of cubic spline functions in space domaine [3].

In this paper, the application of Berenger's PML technique [4] in a two-dimensional MRTD scheme is proposed. The scheme is applied to the calculation of the characteristic impedance of a microstrip transmission line, as well as the field pattern of the first mode. The microstrip geometry analyzed in this paper is shown in Fig.1. The results are compared to data calculated by use of the conventional FDTD technique.

II The 2D-MRTD scheme

For simplicity and without loss of generality, the 2D-MRTD scheme for a homogeneous medium will be presented herein. The derivation is similar to that of Yee's FDTD scheme, which uses the method of moments with pulse functions as expansion and test functions [5]. The magnetic field components are shifted by half a discretization interval in space and time-domain with respect to the electric field components.

Based on [6], Maxwell's H-curl equation for a homogeneous medium with the permittivity ϵ and the permeability μ may be written in the form of the following three scalar equations:

$$\epsilon\frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y} + \beta H_y \quad (1)$$

$$\epsilon\frac{\partial E_y}{\partial t} = -\beta H_x - \frac{\partial H_z}{\partial x} \quad (2)$$

$$\epsilon\frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \quad (3)$$

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where β is the propagation constant. To derive the 2D-MRTD scheme, the field components are represented by a series of cubic spline Battle-Lemarie scaling functions in space and pulse functions in time and are inserted in Maxwell's equations. Afterwards, Maxwell's equations are sampled with pulse functions in time- and scaling functions in space-domain.

As an example, let us consider the discretization of eq.(1). For a homogeneous medium with the permittivity ϵ, sampling $\partial E_x/\partial t$, $\partial H_z/\partial y$ and H_y in space and time gives

$$\frac{\epsilon}{\Delta t}({}_{k+1}E^{\phi x}_{l+1/2,m} - {}_{k}E^{\phi x}_{l+1/2,m}) = \frac{1}{\Delta y}(\sum_{i'=-m_2}^{m_1} a(i')_{k+1/2}H^{\phi z}_{l+1/2,m+i'+1/2}) + \beta \; {}_{k+1/2}H^{\phi y}_{l+1/2,m} \tag{4}$$

where ${}_kE^{\phi x}_{l,m}$, ${}_kH^{\phi z}_{l,m}$ and ${}_kH^{\phi y}_{l,m}$ are the coefficients for the electric and magnetic field expansions. The indices l, m and k are the discrete space and time indices related to the space and time coordinates via $x = l\Delta x, y = m\Delta y$ and $t = k\Delta t$, where $\Delta x, \Delta y$ are the space discretization intervals in x- and y-direction and Δt is the time discretization interval. The coefficients $a(i)$ are given in [1]. Here the summation is truncated to m_1=8 and m_2=9, since higher order $a(i)$ coefficients are negligible.

In the MRTD scheme, it is impossible to implement localized boundary conditions so the perfect electric and magnetic boundary conditions are modelled by use of the image principle. Due to the nature of the Battle-Lemarie expansion functions, the total field in the MRTD technique is a summation of the contributions from the non-localized scaling functions. The field values at the neighboring cells can be combined appropriately by adjusting the scaling functions' values and by applying the image principle [1].

For open structures, the perfectly matched layer (PML) technique [4] can be applied by assuming that the conductivity is given in terms of scaling functions instead of pulse functions with respect to space. The spatial distribution of the conductivity for the absorbing layers is modelled by assuming that the amplitudes of the scaling functions have a parabolic distribution. For the PML absorbing material in the x-direction with ϵ,μ and conductivity σ^E, the term $\sigma^E E_x$ must be added to the left side of eq.(2). Substituting [7]:

$$E_i(x,y,t) = \tilde{E}_i(x,y,t)e^{-\sigma^E_{(x)}t/\epsilon} \tag{5}$$

and

$$H_i(x,y,t) = \tilde{H}_i(x,y,t)e^{-\sigma^H_{(x)}t/\mu} \tag{6}$$

for i=x,y,z, leads to the following equation:

$$\epsilon\frac{\partial \tilde{E}_x}{\partial t} = \frac{\partial \tilde{H}_z}{\partial y} + \beta\tilde{H}_y \quad , \tag{7}$$

which may be discretized in the same way as eq.(1), which is described above.

For all simulations, a parabolic distribution of the conductivity σ is used in the PML region (N cells):

$$\sigma^{E,H}_{(m\Delta x)} = \sigma^{E,H}_{max}(\frac{m}{N})^2 \qquad \text{for m=0,1,..,N,} \tag{8}$$

where $\sigma^{E,H}_{max}$ is the maximum conductivity at the end of the absorbing layer. As in [4], the magnetic conductivity σ^H has to be chosen as:

$$\frac{\sigma^E_{(m\Delta x)}}{\epsilon} = \frac{\sigma^H_{(m\Delta x)}}{\mu} \qquad \text{for m=0,1,..,N,} \tag{9}$$

for a perfect absorption of the outgoing waves.

The MRTD mesh is terminated by a perfect electric conductor (PEC) at the end of the PML region, modelled by applying the image theory. Discretizing eq.(7) and inserting eqs.(5) and (6) yields:

$$\begin{aligned} {}_{k+1}E^{\phi x}_{l+1/2,m} &= e^{-\sigma^E_{(m\Delta x)}\Delta t/\epsilon}\, {}_{k}E^{\phi x}_{l+1/2,m} + \\ &+ \frac{\Delta t}{\epsilon} e^{-\sigma^E_{(m\Delta x)}(\Delta t/2)/\epsilon} \Big(\frac{1}{\Delta y} \sum_{i'=-m_2}^{m_1} a(i)_{k+1/2} H^{\phi z}_{l+1/2,m+i'+1/2} + \beta_{k+1/2} H^{\phi y}_{l+1/2,m} \Big) \end{aligned} \quad . \tag{10}$$

The stability condition for the 2D-MRTD scheme results in

$$\Delta t \leq \frac{1}{1.568c\sqrt{(\frac{1}{\Delta x})^2 + (\frac{1}{\Delta y})^2 + (\frac{\beta}{2})^2}} \quad , \tag{11}$$

where c is the wave propagation velocity. In contrast to Yee's FDTD scheme, it is preferable to choose Δt at least five time less than the stability limit. In this way, much more linearity of the dispersion characteristics is achieved [1].

III Applications of the 2D-MRTD scheme

We apply the 2D-MRTD scheme to the analysis of the microstrip line depicted in Fig.1 for the first (quasi-TEM) propagating mode. The analysis for the higher order propagating modes is straightforward. The central strip has a length of $23.8mm$ and the distances from the top and bottom are $5.5mm$ and $16.5mm$ respectively. The structure is filled with air ($\epsilon_r = 1.$). The PML absorber is applied for 5 cells to the left and the right sides of the structure and the maximum conductivity is $\sigma^E = 0.1$S/m. For the analysis using Yee's FDTD scheme, a 42×28 mesh was used resulting in a total number of 1176 grid points. Analyzing the structure with the 2D-MRTD scheme, a mesh 12×4 (48 grid points) was chosen reducing the total number of grid points by a factor of 24.5 . In addition, the execution time for the analysis was reduced by a factor of 4 to 5. The time discretization interval was chosen to be identical for both schemes and equal to 1/10 of the 2D-MRTD maximum Δt. For the analysis we used $\beta = 30$ and 20,000 time-steps. The frequency of the first propagating mode calculated by use of 2D-MRTD is found to be equal to 1.4325 GHz with error of less than 0.1% in comparison to the theoretical value of 1.4324 GHz. The calculated frequency from the conventional FDTD technique equals to 1.4323 GHz.

In Fig.2, the pattern of the E_y field just below the strip has been calculated and plotted by use of the 2D-MRTD scheme. The pattern obtained by use of the conventional FDTD scheme is plotted for comparison. Since the edge effect is prominent, a mesh 12×8 (96 grid points) was used for the MRTD simulation.

Mode	Analytic values	2D-MRTD	relative error	Yee's FDTD	relative error
Quasi-TEM	1.4324 GHz	1.4329 GHz	0.035%	1.4321 GHz	-0.021%

Table 1: Dominant mode frequencies for $\beta = 30$

IV Conclusion

The PML Absorber has been implemented in the two-dimensional MRTD scheme. The scheme has been applied to the numerical analysis of a microstrip line. The field patterns and the characteristic impedance have been calculated and verified by comparison to reference data. In comparison to Yee's conventional FDTD scheme, the proposed 2D-MRTD scheme offers memory savings by a factor of 24.5 and execution time savings by a factor of about 4–5 while maintaing a comparable accuracy.

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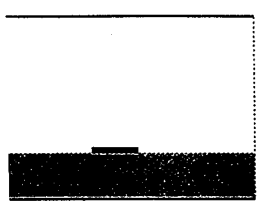

Figure 1: Microstrip structure.

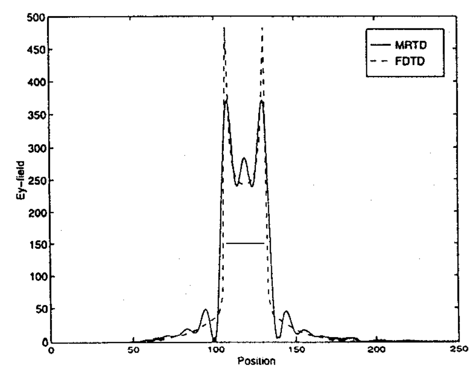

Figure 2: TEM E_y pattern.