

Modeling of Membrane Patch Antennas Using MRTD Analysis

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Abstract

The Multiresolution Time-Domain (MRTD) scheme with perfectly matched layer (PML) absorbing boundaries is applied to the analysis of a membrane patch antenna. The results are compared to those obtained by use of the conventional FDTD technique and substantial reductions in memory requirements are observed.

I Introduction

Recently Multiresolution Time-Domain Analysis has been successfully applied to simulate a variety of microwave structures [1]. MRTD has performed complete analysis of both planar circuits [2] and resonating structures [3]. Additionally conventional FDTD absorbers, such as PML [4] have been generalized in order to analyze open planar structures [5]. In all cases, MRTD has demonstrated a high degree of savings in execution time and memory requirements with respect to FDTD.

In this paper the techniques described are applied to the simulation of a membrane patch antenna with a center frequency of 9 GHz. Full 3D MRTD analysis with PML along three coordinate directions is used to simulate the antenna. The MRTD scheme is applied to the calculation of S-parameters for the membrane antenna and is compared to conventional FDTD.

II Application of PML to the 3D MRTD scheme

To derive the 3D MRTD scheme, the field components in Maxwell's E-curl and H-curl equations are represented by a series of cubic spline Battle-Lemarie scaling functions in space and pulse functions in time. These equations are sampled with pulse functions in time- and scaling functions in space-domain, as detailed in [1]. As an example, consider the discretization of:

$$\epsilon \frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \quad (1)$$

For a homogeneous medium with the permittivity ϵ , expanding and sampling $\partial E_x/\partial t$, $\partial H_z/\partial y$ and $\partial H_y/\partial z$ with scaling and pulse functions in space and time gives

$$\begin{aligned} & \frac{\epsilon}{\Delta t} \left({}_{k+1}E_{l+1/2,m,n}^{\phi x} - {}_kE_{l+1/2,m,n}^{\phi x} \right) \\ &= \frac{1}{\Delta y} \sum_{i=m-9}^{m+8} a(i) {}_{k+1/2}H_{l+1/2,i+1/2,n}^{\phi z} - \frac{1}{\Delta z} \sum_{i=n-9}^{n+8} a(i) {}_{k+1/2}H_{l+1/2,m,i+1/2}^{\phi y} \end{aligned} \quad (2)$$

where ${}_kE_{l,m,n}^{\phi x}$, ${}_kH_{l,m,n}^{\phi z}$ and ${}_kH_{l,m,n}^{\phi y}$ are the coefficients for the electric and magnetic field expansions. The indices l, m, n and k are the discrete space and time indices related to the space and time coordinates via $x = l\Delta x$, $y = m\Delta y$, $z = n\Delta z$ and $t = k\Delta t$, where Δx , Δy , Δz are the space discretization intervals in x-, y- and z-directions and Δt is the time discretization interval. The coefficients $a(i)$ are given in [1].

To derive the perfectly matched layer (PML) technique [4] along one coordinate direction it is assumed that the conductivity is given in terms of scaling functions with respect to space. The spatial distribution of the

conductivity for the absorbing layers is simulated by assuming that the amplitudes of the scaling functions have a parabolic distribution [5]. For the PML absorbing material in the y-direction with ϵ , μ and conductivity σ^E , the term $\sigma_{(y)}^E E_x$ must be added to the left side of eq.(1). Then, substituting the following into eq.(1):

$$E_i(x, y, z, t) = \tilde{E}_i(x, y, z, t)e^{-\sigma_{(y)}^E t/\epsilon} \quad (3)$$

$$H_i(x, y, z, t) = \tilde{H}_i(x, y, z, t)e^{-\sigma_{(y)}^H t/\mu}, \quad (4)$$

and assuming that the PML is only along the y-direction leads to the following equation:

$$\begin{aligned} {}_{k+1}E_{i+1/2,m,n}^{\phi x} &= e^{-\sigma_{(m\Delta y)}^E \Delta t/\epsilon} {}_k E_{i+1/2,m,n}^{\phi x} + \frac{\Delta t}{\epsilon} e^{-\sigma_{(m\Delta y)}^E (\Delta t/2)/\epsilon} \left(\frac{1}{\Delta y} \sum_{i=m-9}^{m+8} a(i) {}_{k+1/2} H_{i+1/2,i+1/2,n}^{\phi z} + \right. \\ &\quad \left. + \frac{1}{\Delta z} \sum_{i=m-9}^{m+8} a(i) {}_{k+1/2} H_{i+1/2,m,i+1/2}^{\phi y} \right) \end{aligned} \quad (5)$$

For all simulations, a parabolic distribution of the conductivity σ is used in the PML region (N cells):

$$\sigma_{(m\Delta y)}^{E,H} = \sigma_{max}^{E,H} \left(\frac{m}{N} \right)^2 \quad \text{for } m=0,1,\dots,N, \quad (6)$$

where $\sigma_{max}^{E,H}$ is the maximum conductivity at the end of the absorbing layer. As in [4], the magnetic conductivity σ^H has to be chosen as:

$$\frac{\sigma_{(m\Delta y)}^E}{\epsilon} = \frac{\sigma_{(m\Delta y)}^H}{\mu} \quad \text{for } m=0,1,\dots,N, \quad (7)$$

for a perfect absorption of the outgoing waves. The MRTD mesh is terminated by a perfect electric conductor (PEC) at the end of the PML region, modelled by applying the image theory.

While the above derivation is adequate for a structure which only needs to be terminated with PML along one direction, such as a shielded thru-line, structures such as patch antennas need PML termination in all three coordinate directions. In this case, the derivation discussed above needs to be extended to three dimensions. The procedure is straightforward and results in the following equation:

$$\begin{aligned} {}_{k+1}E_{i+1/2,m,n}^{\phi x} &= e^{-\sigma_{(i\Delta x)}^E \Delta t/\epsilon} e^{-\sigma_{(m\Delta y)}^E \Delta t/\epsilon} e^{-\sigma_{(n\Delta z)}^E \Delta t/\epsilon} {}_k E_{i+1/2,m,n}^{\phi x} + \\ &\quad + \frac{\Delta t}{\epsilon} e^{-\sigma_{(i\Delta x)}^E (\Delta t/2)/\epsilon} e^{-\sigma_{(m\Delta y)}^E (\Delta t/2)/\epsilon} e^{-\sigma_{(n\Delta z)}^E (\Delta t/2)/\epsilon} \left(\frac{1}{\Delta y} \sum_{i=m-9}^{m+8} a(i) {}_{k+1/2} H_{i+1/2,i+1/2,n}^{\phi z} + \right. \\ &\quad \left. + \frac{1}{\Delta z} \sum_{i=m-9}^{m+8} a(i) {}_{k+1/2} H_{i+1/2,m,i+1/2}^{\phi y} \right) \end{aligned} \quad (8)$$

As in eq.(6) a parabolic distribution for the conductivity is applied in the x-, y- and z-directions.

III Applications of the 3D-MRTD scheme

The object of this paper is to apply the 3D MRTD scheme to the analysis of membrane patch antennas. However, in order to test the application of PML to the 3D MRTD scheme, a microwave thru-line is analyzed using MRTD and FDTD. The thru-line has a width of 0.4 mm and length of 10.0 cm and is placed in the center of a cavity with dimensions 1.6mm \times 10.0cm \times 1.6mm. A Gaussian pulse is used to excite the thru-line

with $f_{max} = 50GHz$ [6] and is placed in the middle of the center conductor 9 mm from the PML layer along the y -axis. The FDTD analysis uses $16 \times 100 \times 16$ mesh while the MRTD analysis uses a $8 \times 20 \times 8$ mesh. Additionally six cells of PML are used along the y -direction at either end of the thru-line with a $\sigma_{max}^{E_y} = 3.0$. Therefore the total discretization of the thru line is $16 \times 112 \times 16$ for FDTD and $8 \times 32 \times 8$ for the MRTD scheme, resulting in a factor of 14.0 savings in memory. The time discretization interval for the MRTD scheme is $\Delta t = 3.92 \cdot 10^{-14}s$, while the FDTD scheme has $\Delta t = 6.335 \cdot 10^{-14}s$. In both cases, the simulation is performed for 6000 time steps. A comparison plot of time vs. E_z -field amplitude is shown in Figure 1. Note that the amplitude of the Gaussian has been normalized and the time-steps multiplied by a constant factor in order to compare the two plots more easily. The initial Gaussian pulse has been completely absorbed by the PML layer along the y -direction.

The membrane patch antenna shown in Figure 2 is simulated using 3D MRTD and FDTD. A full description of the parameters of the antenna can be found in [7]. A PML layer of six cells is used along the $\pm x$, $\pm y$ and $\pm z$ directions, resulting in an FDTD mesh of $72 \times 112 \times 28$ and a MRTD mesh of $42 \times 62 \times 12$, a factor of 7.22 savings in memory. In the PML layers $\sigma_{max}^{E_x} = \sigma_{max}^{E_y} = \sigma_{max}^{E_z} = 3.0$ for FDTD and MRTD. The time discretization interval used for the MRTD scheme is $\Delta t = 1.6008 \cdot 10^{-13}s$ while the FDTD time discretization interval is $\Delta t = 1.3297 \cdot 10^{-13}s$. In both cases the simulation is performed for 7000 time steps. The antenna feed line is 20 mm long and the Gaussian pulse is sent from a point $y=4$ mm from the edge of the PML layer in the FDTD and MRTD simulations. Figure 3 shows a plot of E_z field values vs. time for MRTD and FDTD. Measurement of the initial and reflected normalized Gaussian pulses occurred at $y = 14$ mm from the edge of the PML layer. Figure 4 shows a plot of the calculated S_{11} [7] for the membrane patch antenna. Note that excellent correlation is achieved between FDTD and MRTD results.

IV Conclusion

A membrane patch antenna is successfully simulated using the 3D MRTD scheme with PML absorber along the x -, y - and z -directions. With respect to calculated S_{11} results MRTD shows excellent correlation with FDTD while exhibiting a memory savings with a factor of 7.22.

V Acknowledgements

This work has been partially funded by NSF and ARL.

References

- [1] M. Krumpholz and L.P.B. Katehi, "MRTD: New Time Domain Schemes Based on Multiresolution Analysis", IEEE Trans. Microwave Theory and Techniques, vol. 44, no. 4, pp. 555-561.
- [2] E. Tentzeris, M. Krumpholz and L.P.B. Katehi, "Application of MRTD to Printed Transmission Lines", Proc. MTT-S 1996, pp. 573-576.
- [3] R. Robertson, E. Tentzeris, M. Krumpholz and L.P.B. Katehi, "Application of MRTD Analysis to Dielectric Cavity Structures", Proc. MTT-S 1996, pp. 1840-1843.
- [4] J.P. Berenger, "A Perfectly Matched Layer for the Absorption of Electromagnetic Waves", J. Comput. Physics, vol. 114, pp. 185-200, 1994.
- [5] E. Tentzeris, R. Robertson, M. Krumpholz and L.P.B. Katehi, "Application of the PML Absorber to the MRTD Technique", Proc. AP-S 1996, pp. 634-637.
- [6] X. Zhang and K.K. Mei, "Time-Domain Finite Difference Approach to the Calculation of the Frequency-Dependent Characteristics of Microstrip Discontinuities", IEEE Trans. Microwave Theory and Techniques, vol. 36, no. 12, pp. 1775-1787.
- [7] D. Sheen, S. Ali, M. Abouzahra and J. Kong, "Application of the Three-Dimensional Finite-Difference Time-Domain Method to the Analysis of Planar Microstrip Circuits", IEEE Trans. Microwave Theory and Techniques, vol. 38, no. 7, pp. 849-856.

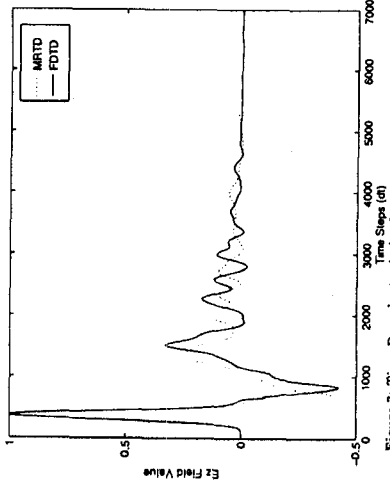


Figure 1: Time-Domain Analysis of a Microstrip Thru-Line.

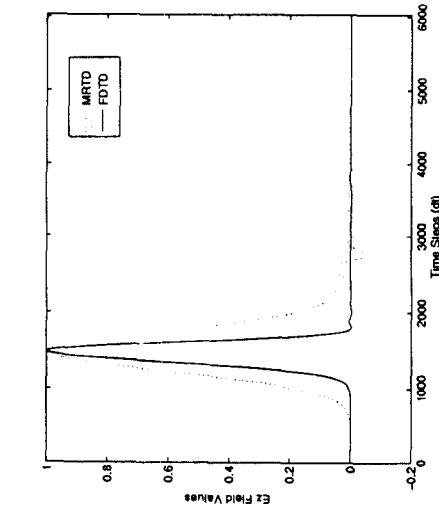


Figure 2: Membrane Patch Antenna.

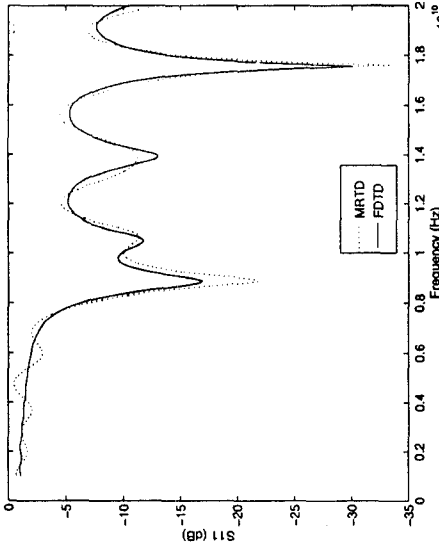


Figure 3: Time Domain Analysis of a Membrane Patch Antenna.

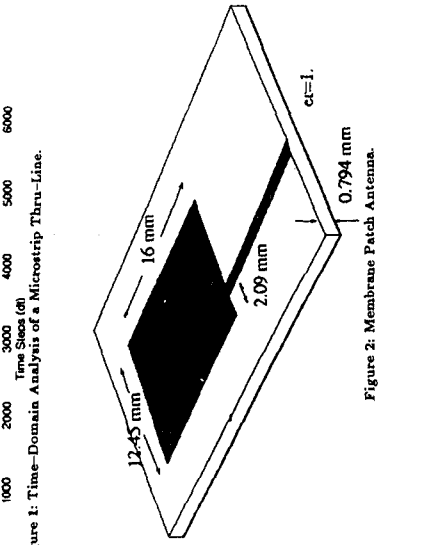


Figure 4: Simulated S11 results for the Patch Antenna.