

# Modeling and Design of RF MEMS Structures Using Computationally Efficient Numerical Techniques

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## Abstract

The modeling of MEMS structures using MRTD is presented. Many complex RF structures have been inadequately studied due to limitations in simulation methods. The space and time adaptive grid, as well as the ability to handle intracell feature variations, makes MRTD an ideal method for modeling these structures. MRTD is shown to be able to handle the complex structures found in modern wireless and microwave communication systems efficiently and accurately. Specifically, micromachined structures such as MEMS are treated.

## Introduction

The rapid advance of technology in the RF/Wireless communications industry is producing new devices faster than they can be accurately and fully characterized. This leads to the production of devices that work for a specific application but are difficult to modify for other uses. This is true in a number of specific areas, such as packaging, MMIC design, and MEMS devices.

MEMS (microelectromechanical systems) devices are a very important field of study for many reasons. The applications range from small embedded sensors to micro-mirrors. In the RF field, much interest is in variable passive components. For these components, the chief characteristics that make MEMS devices desirable are low loss and parameter variability.

The three main causes of loss in RF devices are metal loss, radiation loss, and substrate loss. The micromachining procedures used in the creation of MEMS devices remove substrate from the structure. With significantly less substrate in MEMS devices, more of the electromagnetic field is contained in the air between membranes. This significantly reduces loss.

An example of a variable passive device is a variable capacitor. Existing variable capacitors, or varactors, are semiconductor devices. As such, they experience many of the noise and distortion problems of active components. MEMS capacitors are similar in form to passive capacitors. However, the plates in a MEMS capacitor move. Thus, MEMS capacitors can be used as variable components with fewer of the negative characteristics of their semiconductor counterparts.

Packaging, MMICs, and MEMS share a number of similarities when considered from a modeling viewpoint. All have complex structures that can be difficult to model. Specifically, small feature size compared to total device size is a limiting issue. In addition, a full-wave simulation is often desired to determine the response of the structure over a

number of bands (operating band, image frequency, LO frequency, baseband, etc.). Thus, a need exists to find a way to model these devices quickly and accurately.

Many current commercial modelers have difficulty handling the complex three-dimensional geometries that appear in modern RF components. In addition, MEMS devices can change their configuration during use. The transient effects of these changes cannot be modeled with frequency domain modeling techniques. Furthermore, the interaction of an excitation with a moving RF structure can cause time domain instabilities. In order to model these new devices, modeling methods that can simulate complex structures with time changes must be applied.

One method that could be used to model these devices is the finite-difference time-domain (FDTD) technique. FDTD is a full-wave modeler, and has the ability to model complex geometries. It is a time domain technique, and could conceivably be modified to handle moving components. However, its dependence on small cell sizes and its difficulties in handling fine resolution limit its uses in the modeling of MEMS devices.

The multiresolution time-domain (MRTD) technique directly addresses many of these limitations. It can model intracell discontinuities and uses an adaptive grid. These features allow MRTD to handle complex structures with significant economies in memory and execution time when compared to FDTD. It is also a time domain technique, and can be modified to accommodate in-simulation changes in structure. In addition, it is a full-wave simulator; it can provide results over any band.

This paper presents how the MRTD method can be applied to model MEMS structures, both static and dynamic. Motion of MEMS structures can be combined into the MRTD electromagnetic model to provide a time domain simulation which models both the EM response of the device and its motion. This can be very useful when modeling devices such as MEMS variable capacitors and switches.

## MRTD Using Haar Basis Functions

The complex features of many devices make them difficult to model. Commercial modelers have several limitations when applied to nonsymmetrical three-dimensional structures. In addition, they cannot model structures with time variation. These characteristics suggest the need to use a full-wave time-domain simulator that can model arbitrary structures.

The FDTD technique has been successfully applied to many structures [1,2]. It is a time-domain technique, and can be expanded to model variations in structure. The two main advantages of the FDTD technique are its ability to model arbitrary structures and its ability to model over a wide

frequency band. FDTD has many disadvantages, however, when applied to some complex structures.

In the FDTD scheme, it is imperative that the cell size be significantly smaller than the maximum wavelength appearing in the simulation. Typically, the cell size must be smaller than one-tenth of the wavelength [2]. This is well below the Nyquist limit. In addition, discontinuities within a cell can be very difficult to model accurately. In a MEMS or packaging structure, limiting the cell size to match the smallest feature can lead to extremely large computational grids. Simulations using these grids may have execution times that prohibit their implementation.

The MRTD technique uses a wavelet and scaling function discretization of Maxwell's equations to model electromagnetic phenomena [3,4]. Together, the wavelet and scaling functions form a complete function space in which the electromagnetic fields can be represented. Conventional FDTD is equivalent to simulating using pulse basis functions only. The wavelet functions, used in MRTD, provide increased resolution and the ability to model intracell discontinuities.

The complete space is made up of an infinite number of wavelets. Typically, in an electromagnetic simulation, the resolution is limited to a level above which wavelet values are negligible. Wavelet functions have several properties that make them well suited to representing functions found in differential equations. The inner product relationships that exist between wavelets of different resolutions, as well as the scaling functions, can be used with the method of moments to create a discretized version of Maxwell's equations. These equations also all have an infinite level of wavelets. Using the limiting technique mentioned above, the simulation can be restricted to a finite number of wavelets. All cells in the simulation need not necessarily use the same number of wavelets, leading to the term multiresolution.

The wavelet limiting process can be further expanded. It is not required that the highest wavelet resolution used be determined prior to simulation. Moreover, the wavelets resolution used in a given cell does not need to be the same during the entire simulation. The wavelet functions can be evaluated at each time step to determine whether they have a significant contribution. Wavelets that do not contribute significantly can be discounted until they do. By using a thresholding algorithm, wavelets can be adaptively added and removed during the simulation. During periods of high field variation, the resolution can be increased. Likewise, it can be decreased during periods of low field variation. This technique has been called the MRTD adaptive grid [5].

Wavelet functions that complement the pulse basis functions used in the Yee FDTD technique form the Haar expansion basis [6]. The Haar basis functions are convenient to use in multiresolution analysis because of their compact support. Haar functions are not as efficient of a representation as other wavelets, however, they provide a great deal of improvement when compared to FDTD alone. In addition, because of their compact support, the use of image theory in the modeling of boundary conditions is avoided. The scaling

function and 0<sup>th</sup> and 1<sup>st</sup> order Haar Wavelet functions are shown in Fig. 1.

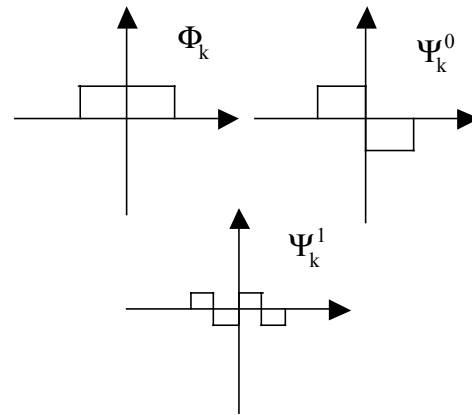


Fig. 1 Scaling function and 0<sup>th</sup> and 1<sup>st</sup> order Haar Wavelets

In a function expansion, only one resolution of scaling function is used. Wavelet functions of an equal level of resolution and higher are used. This is because scaling functions are orthogonal to all wavelets of higher resolution. There are several other important orthogonality relationships involving wavelets [7,8].

The other important aspect of modeling using Haar wavelets is the ability to model arbitrarily placed PECs. This is because of wavelet characteristics that become important when wavelet resolutions are increased. For every increase in the order of a finite-domain wavelet, the support becomes half as large. Thus, for each increase in resolution, twice as many wavelets are needed to model the same domain. By increasing the resolution, wavelets can be found that have boundaries at any arbitrary points. This characteristic of wavelet decomposition is quite useful when a PEC intersects a cell.

When using infinite-domain wavelets, such as the popular Battle Lemarie functions [7], perfect electrical conductors (PECs) must be modeled through the use of image theory [3-5]. This can lead to a very complex implementation. However, PECs can be very naturally implemented into a simulation using finite-domain wavelets, such as the Haar basis functions [6].

The implementation of PECs using Haar wavelets is demonstrated in Fig. 2. In the figure, a PEC intersects a cell. Because the scaling function and zero resolution wavelet intersect the PEC, their magnitude is set to zero. The first resolution wavelet has a boundary on the PEC. This naturally enforces the PEC boundary condition, namely that the tangential field on the PEC be zero, because the value of the wavelets at their boundaries is zero. Thus, this wavelet and all wavelets of higher resolution can be used to model the fields in the cell. Because the resolution can be increased to place a wavelet boundary at an arbitrary location within a cell, a PEC can be placed anywhere within a cell. The PEC will be modeled by placing a wavelet boundary sufficiently close to the PEC interface. In a static simulation, the wavelet resolution needed to model a PEC can be determined a-priori,

and memory can be saved by not allocating storage for lower resolution wavelets.

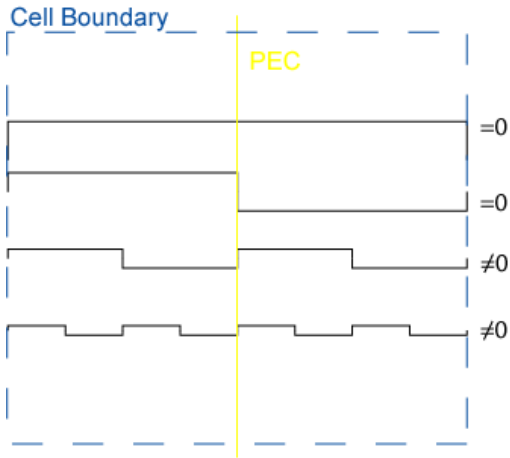


Fig. 2 Intersection of cell with PEC using Haar Wavelets

### Modeling of MEMS Motion

The most distinct feature of MEMS devices is their movement. Other structures that appear in the analysis of RF/wireless components have complex structures of varying sizes, but the movement in MEMS devices creates a unique modeling problem. MEMS devices have membranes that can change position. Usually this movement is caused externally. However, the membranes in MEMS devices are also affected by their internal electromagnetic fields. In this way it is natural to combine an electromagnetic simulator and a motion model in order to characterize a MEMS device.

MEMS devices come in several classifications. A simple example is used here to demonstrate the complexity of combining a motion simulation with an MRTD simulation. The devices considered herein are electrostatically-actuated parallel-plate variable capacitors. While several complex mechanisms exist for their motion, most of the important aspects of their movement can be represented through a one dimensional spring mass model [9]. Fig. 3 is a diagram of this model.

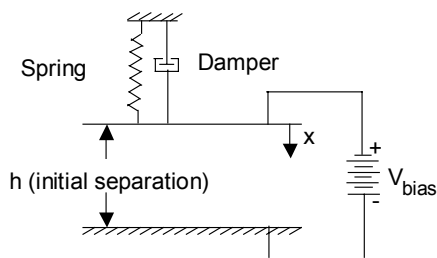


Fig. 3. MEMS Parallel Plate Capacitor Model

In Fig. 3, the spring and damper restrain the top plate; the bottom plate is fixed. When the bias voltage is applied, the top plate is attracted to the bottom plate, which is resisted by the spring and damper. When the capacitor is in use the top

plate has a force due to the incident RF field in addition to the applied bias.

In the actual capacitor, the plate is not constrained by a miniature spring. Instead, it is supported by a column of substrate. The substrate beam can be modeled as a spring which contracts in response to the applied force. The damper appears in the diagram to account for the effect of air resistance on the moving plate. The known characteristics of the substrate and air resistance can be used to determine the parameters of the components in the model. This motion can be represented by a second order differential equation.

The differential equation used to model the parallel plate capacitor can be discretized using central differences much the same way that the electromagnetic fields are discretized in FDTD. The resulting equation:

$$x^{n+1} = 2x^n \left( \frac{2m - k\Delta t^2}{2m + b\Delta t} \right) + x^{n-1} \left( \frac{b\Delta t - 2m}{b\Delta t + 2m} \right) + \frac{2\Delta t^2 \epsilon_0 AV^2}{(2m + b\Delta t)(x^n - h)^2} \quad (1)$$

gives the position of the plate in time.

The above equation uses the standard notation:

$$u(n\Delta t) = u^n \quad (2)$$

In (1),  $k$  represents the spring constant,  $b$  is the damping coefficient,  $A$  is the area of the plates, and  $m$  is the mass of the top plate.

The position of the top plate is given at all time by (1). In order to couple the RF excitation with the applied bias,  $V$  can be changed to  $V(t)$  to represent the summation of the bias and the excitation. It is important to note that  $x$  is a continuous variable, that is it can take any value. This is unlike FDTD and MRTD, where space is discretized. In both simulations, however, time is discretized.

### Applying MRTD to MEMS

The previously summarized features, adaptive gridding and arbitrary metal placement, make MRTD an excellent method for modeling MEMS devices. Using these features, simulations that would be very difficult in other methods can be completed. The wavelets used in multiresolution analysis allow for structures with large and varying compositions to be modeled effectively. In addition, these features allow for structures with time varying geometries to be modeled.

MEMS devices often have large feeding or connecting structures that are attached to the fine features of the actual MEMS device. To accurately model the MEMS device, the entire structure must be considered. If the same resolution is to be used across the entire simulation, the computational grid will be obscenely large. However, the resolution need not, and usually is not, the same across the entire structure.

In MRTD, a coarse grid can be applied to the entire structure, and the adaptive grid can provide increased resolution in areas of complex structure. The cells need not be laid out with respect to PEC or dielectric location. In this manner, feeding structures can have very low resolution compared to the overall structure. In addition, as the simulation progresses, the resolution will automatically increase in areas of high field variation.

If the capacitor above was to be modeled in MRTD, the only needed information would be the position of the PECs and the dielectrics. Due to the fact that MRTD is a time-domain simulator, moving structures can also be modeled. The movement of the plates in time represents changing boundary conditions in the MRTD simulation.

To combine the simulations, they have to be run simultaneously. Both are time domain; quantities needed by one simulation are simply accessed by the other. Using the capacitor simulation as an example, when performing an update on the position of the top plate in the motion simulation, the field information from the electromagnetic simulation is used to determine the voltage between the plates (in addition to the DC bias voltage). When the electromagnetic simulation is next updated, the new position of the top plate is used to determine the electromagnetic fields. The time constants between the two simulations are different, meaning that the motion simulation need only be updated every few hundred of the electromagnetic simulation time steps.

The difficulty in combining the simulations is modeling the changing structure. In FDTD, this would indeed be difficult. The grid used would have to be changed at each update of the top plate movement in order to have the PEC exist on the edge of a cell. This could either be accomplished by using an adaptive FDTD grid or a very fine mesh in the area of the top plate. Both of these solutions would be inaccurate and would result in significant computation time.

In MRTD, an adaptive grid already exists. The moving top plate can be modeled as a moving PEC. To represent the position of the plate, the resolution is set to a level where the wavelet intersects with the edge of the PEC. In this way, the non-discrete motion of the top plate can be easily accounted for.

The variable gridding that makes it possible to model motion of the capacitor plate also makes it possible to study the capacitor within a larger structure. The capacitor is very small compared to a feeding line or structure. Using the variable resolution, the surrounding structure can be modeled with an appropriately coarser grid and still provide an accurate simulation.

## Conclusions

A method for modeling MEMS devices using MRTD was presented. It was shown that many of MRTD's hallmark capabilities provide an already rich toolkit for the modeling of MEMS devices. MRTD can model changes in structure over time because it is a time domain simulator. In addition, the variable grid and the ability to arbitrarily place metals in a structure allow for the modeling of a structure that changes configuration during the simulation. Furthermore, the MRTD method can be used to determine the interaction of devices, even complex ones such as MEMS, with their surrounding environment or package. This is becoming increasingly important in the modeling of modern structures because as frequency increases and size decreases, package device interaction is becoming more pronounced.

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