APPLICATION OF MRTD TO PRINTED TRANSMISSION LINES

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ABSTRACT

The recently developed MRTD scheme is applied to the solution of printed transmission line problems. Specifically, the field pattern and the characteristic impedance of a membrane stripline is calculated. The results are compared to those obtained by use of the conventional FDTD scheme to indicate considerable savings in memory and computational times.

INTRODUCTION

Significant attention is being devoted now-a-days to the analysis and design of various types of transmission lines. The finite-difference-time-domain (FDTD) scheme is one of the most powerful numerical techniques used in printed lines either to calculate the propagation constant and characteristic impedance or to probe the patterns of the excited electromagnetic fields. However, despite its simplicity and modeling versatility, the FDTD scheme suffers from serious limitations due to the substantial computer resources required to model electromagnetic problems with medium or large computational volumes.

To alleviate this problem, a multiresolution time domain (MRTD) scheme has been proposed [1], [2]. It has been shown that Yee's FDTD scheme can be derived through the application of the method of moments for the discretization of Maxwell's equations [3] with pulse basis functions for the expansion of the unknown fields. The use of scaling and wavelet functions as a complete set of basis functions is called multiresolution analysis and leads to the MRTD Schemes. These schemes not only improve the accuracy in solution compared to conventional FDTD, but also result in significant reductions of computer resources.

In this paper, a 2D version of multiresolution time domain (MRTD) scheme is presented. This scheme is applied to the calculation of the propagation constant and characteristic impedance of a membrane stripline (see Fig.1) [4]. The same scheme is applied to compute the field patterns of the first two membrane stripline modes. The results are compared to data calculated by use of the conventional FDTD scheme.

THE 2D-MRTD SCHEME

For simplicity in the presentation and without loss of generality, the 2D-MRTD scheme for a homogeneous medium will be presented herein. The derivation is similar to that of Yee's FDTD scheme, which uses the method of moments with pulse functions as expansion and test functions. The magnetic field components are shifted by half a discretization interval in space and time domains with respect to the electric field components.

For a homogeneous medium with the permittivity ϵ and the permeability μ, Maxwell's H-curl equation

$$\nabla \times \mathbf{H}' \quad = \quad \epsilon \frac{\partial \mathbf{E}'}{\partial t} \tag{1}$$

may be written in the form of three scalar equations. Based on [5], we assume that the electric and magnetic field components can be written as

$$E_x^{'}, E_y^{'}, H_z^{'} = [E_x(x,y,t), E_y(x,y,t), H_z(x,y,t)] j e^{-j\beta z} \tag{2}$$

$$H_x^{'}, H_y^{'}, E_z^{'} = [H_x(x,y,t), H_y(x,y,t), E_z(x,y,t)] e^{-j\beta z} \tag{3}$$

where β is the propagation constant. In view of the above field representation, the three scalar Maxwell's equations can be written as

$$\epsilon\frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y} + \beta H_y \quad (4)$$

$$\epsilon\frac{\partial E_y}{\partial t} = -\beta H_x - \frac{\partial H_z}{\partial x} \quad (5)$$

$$\epsilon\frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \quad . \quad (6)$$

To derive the 2D-MRTD scheme, the field components are represented by a series of cubic spline Battle-Lemarie [6] scaling functions in space and pulse functions in time. After inserting the field expansions in Maxwell's equations, we sample them using pulse functions in time- and scaling functions in space-domain.

As an example, let's consider the discretization of eq.(4). Sampling $\partial E_x/\partial t$, $\partial H_z/\partial y$, H_y in space and time, the following difference equation is obtained for a homogeneous medium with the permittivity ϵ,

$$\frac{\epsilon}{\Delta t}({}_{k+1}E^{\phi x}_{l+1/2,m} - {}_{k}E^{\phi x}_{l+1/2,m}) = \frac{1}{\Delta y}(\sum_{i=m-m_2}^{m+m_1} a(i)_{k+1/2}\, H^{\phi z}_{l+1/2,i+1/2}) + \beta\ {}_{k+1/2}H^{\phi y}_{l+1/2,m} \quad , \quad (7)$$

where ${}_kE^{\phi\kappa}_{l,m}$ and ${}_kH^{\phi\kappa}_{l,m}$ with $\kappa = x, y, z$ are the coefficients for the electric and magnetic field expansions. The indices l, m and k are the discrete space and time indices, which are related to the space and time coordinates via $x = l\Delta x, y = m\Delta y$ and $t = k\Delta t$, where $\Delta x, \Delta y$ are the space discretization intervals in x- and y-direction and Δt is the time discretization interval. The coefficients $a(i)$ are derived and given in [2]. The summation constants m_1, m_2 depend on the expected accuracy in the calculations. For an accuracy of 0.1% the values $m_1 = 8$ and $m_2 = 9$ have been used. A similar approach is followed for the discretization of the rest of Maxwell's equations.

Due to the nature of the Battle-Lemarie expansion functions, the total field is a summation of the contributions from the non-localized scaling functions. For example, the total electric field $E_x(x_o, y_o, t_o)$ with $(k - 1/2)\,\Delta t < t_o < (k + 1/2)\,\Delta t$ is calculated in the same way with [2] by

$$E_x(x_o, y_o, t_o) = \sum_{l',m'=-l_1}^{l_1} {}_kE^{\phi x}_{l'+1/2,m'}\, \phi_{l'+1/2}(x_o)\, \phi_{m'}(y_o) \quad (8)$$

where $\phi_m(x) = \phi(\frac{x}{\Delta x} - m)$ represents the Battle-Lemarie scaling function. Practically, the above summation is truncated to very few terms due to the exponentially decaying support of the scaling functions. For an expected accuracy of 0.1% the value of $l_1 = 4$ was used in the simulations.

Extending the dispersion analysis of [2] from 3- to 2-dimension space, the stability condition for the 2D-MRTD scheme results in

$$\Delta t \leq \frac{1}{1.568c\sqrt{(\frac{1}{\Delta x})^2 + (\frac{1}{\Delta y})^2 + (\frac{\beta}{2})^2}} \quad (9)$$

with the wave propagation velocity c.

APPLICATIONS OF THE 2D-MRTD SCHEME

The 2D-MRTD scheme is applied to the analysis of the membrane stripline of Fig.1 for the first two propagating modes. The analysis for the higher order propagating modes is straightforward. For simplicity in the simulation and without loss of accuracy, we approximate this geometry with the rectangular shielded stripline of Fig.2. For the analysis using Yee's FDTD scheme, a 20×10 mesh was used resulting in a total number of 200 grid points. When the structure was analyzed with the 2D-MRTD scheme, a mesh 4×2 (8 grid points) was chosen reducing the total number of grid points by a factor of 25. In addition, the execution time for the analysis was reduced by a factor of 4 to 5. The time discretization interval was chosen to be identical for both schemes and equal to the 1/5 of the 2D-MRTD maximum Δt. For the analysis we have chosen $\beta = 30$ and 8,000 time-steps.

From Table 1 it can be observed that the calculated frequencies of the two first propagating modes from 2D-MRTD scheme are very close to the theoretical values. The use of non-localized basis functions in the 2D-MRTD scheme causes significant effects. Localized boundary conditions are impossible to be implemented, so the perfect electric boundary conditions are modelled by use of the image principle in a generic way. This implementation of the image theory is performed automatically for any number of PEC, PMC boundaries.

The field values at the neighboring cells can be combined appropriately by adjusting the scaling functions' values and by applying the image principle. For example, the vertical-to -strip E_y field at the position (x,y) for a specific k-time-step can be calculated from eq.(8) by simply truncating the l,m summation from -4 to 4 for each index. That means that the summation based only at the 4 neighboring cells from each side gives the total field component values with good accuracy. The MRTD can calculate the fields locally with high accuracy while it still provides substantial savings in memory and time. Specifically, the agreement of the MRTD calculated field pattern with the reference data is very good for the 1st higher order mode (shield TE_{10}), where the field values are changing slowly (sinusoidally). For the TEM mode where the edge effect is more prominent, more expansion functions are needed for the accurate reconstruction of the field locally. It can be observed from Fig.3 - E_y field distribution just below the strip - that the accuracy of the representation is improved when the number of scaling functions is increased from 4 to 8 along the strip direction and from 2 to 4 along the normal to the strip direction. The pattern obtained by use of the conventional FDTD scheme has been plotted for comparison. In Fig.4, the value of E_y field for the geometry cross-section has been calculated and plotted by use of the 2D-MRTD scheme.

To calculate the characteristic impedance Z_o for the TEM mode of the stripline, we use the following equation:

$$Z_o = \frac{V}{I} = \frac{\int_{C_v} E_y dy}{\oint_{C_c} H dl} \quad , \qquad (10)$$

where the integration paths C_v and C_c are shown in Fig.2. Since both of the schemes used in the analysis are discrete in space-domain, the above integrals are transformed to summations. It can be observed from the Table 2 that the accuracy of the calculation of the Z_o by use of the MRTD is much better than that of the Yee's FDTD scheme with a 20x10 mesh (relative error -3.06%). The oscillating values of Z_o within 0.5Ω by use of the MRTD are due to the fact that the very small relative error is not determined by the discretization any more, but by the numerical errors of the MRTD code.

CONCLUSION

A multiresolution time-domain scheme in 2 dimensions has been proposed and has been applied to the numerical analysis of a stripline. The field patterns and the characteristic impedance have been calculated and verified by comparison to reference data. In comparison to Yee's conventional FDTD scheme, the proposed 2D-MRTD scheme offer memory savings by a factor of 25 and execution time savings by a factor of about 4-5 maintaing a better accuracy. These savings are consistent with results reported in [1], [2] for 3D-MRTD.

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Table 1: Propagating mode frequencies

Mode	TEM	Shield TE_{10}
Analytic values	1.4324 GHz	3.4615 GHz
4x2 MRTD	1.4325 GHz	3.4648 GHz
Rel.Error	0.007%	0.095%
4x4 MRTD	1.4325 GHz	3.4641 GHz
Rel.Error	0.007%	0.075%
8x4 MRTD	1.4325 GHz	3.4633 GHz
Rel.Error	0.007%	0.052%
20x10 FDTD	1.4322 GHz	3.4585 GHz
Rel.Error	-0.014%	-0.087%

Table 2: Z_o calculated by 2D-MRTD

	Z_o (Ω)	Relative error
Analyt. Value [7]	61.03	0.0%
4x2 MRTD	61.17	+0.23%
4x4 MRTD	61.44	+0.67%
8x4 MRTD	61.22	+0.31%
20x10 FDTD	59.16	-3.06%

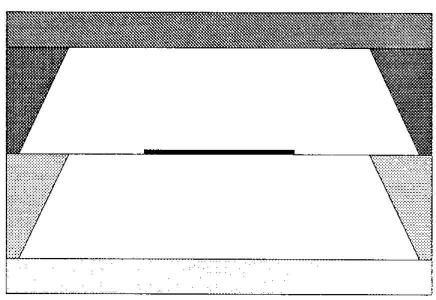

Figure 1: Membrane Stripline.

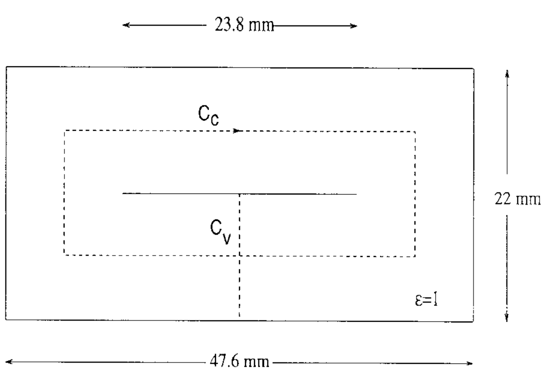

Figure 2: Stripline Geometry.

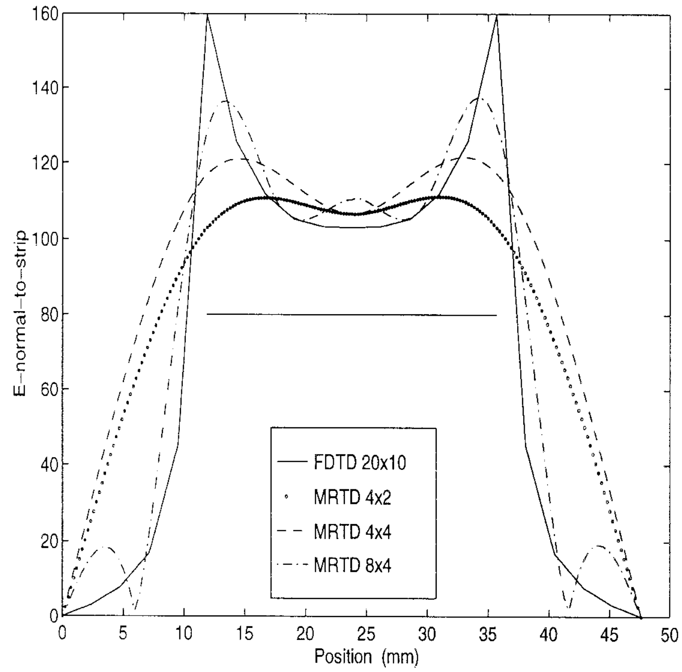

Figure 3: TEM E-pattern.

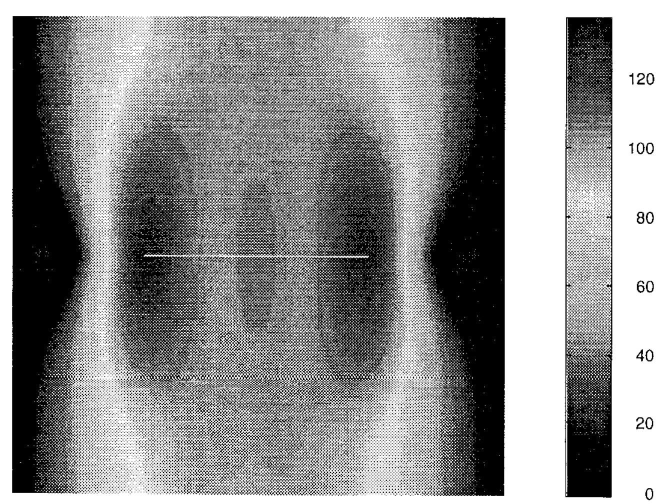

Figure 4: TEM 2D E-pattern.