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RF MEMS: DEVELOPMENT OF DESIGN RULES

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ABSTRACT

The modeling of MEMS passive electrical components is presented. The devices are modeled using the MRTD (multiresolution time-domain) and FDTD (finite-difference time-domain) electromagnetic modeling techniques. Methods are presented that allow these time-domain electromagnetic models to be combined with time-domain motion models of MEMS devices.

INTRODUCTION

MEMS devices have several characteristics that make them attractive to use as circuit components. Their low loss characteristics, as well as variability, make them unique among currently available technology. However, regardless of their gains, the effects of design choices on the performance of the device are largely unknown. One reason for this is that the devices are difficult to model. MEMS electrical components have both electromagnetic and mechanical interaction, meaning that a simulator for these devices must be able to model both phenomena.

The multi-resolution time-domain (MRTD) and finite-difference time-domain (FDTD) techniques have been successfully used to model the electromagnetic characteristics of many devices. MRTD has several advantages over FDTD that make it the method of choice for modeling complex structures such as MEMS, the most important being the adaptive grid provided by wavelet analysis. This paper outlines a technique in which these time-domain methods can be combined with a time-domain motion model of MEMS devices. This is done using the example of a MEMS variable capacitor. This MEMS simulator can be used to determine the performance of MEMS devices prior to fabrication, and thus aid in the development of MEMS design rules.

FDTD AND MRTD BACKGROUND

The FDTD electromagnetic modeling technique [1] utilizes a finite difference discretization of Maxwell's electromagnetic equations to numerically model the electromagnetic interaction of structures. It has been used to simulate a variety of geometries and determine the characteristics of a variety of structures [2]. There are, however, several well-known limitations of the FDTD technique. The FDTD scheme models a structure by creating a discrete grid that represents the fields on the structure. The most constraining requirement of the FDTD scheme is that all cell dimensions must be at most one tenth of the maximum wavelength that will be used in the structure [2]. This limitation makes the modeling of high-frequency structures very difficult.

The MRTD scheme utilizes a scaling and wavelet function discretization of the electromagnetic fields, as opposed to the pulse scaling functions employed by the Yee FDTD technique. The wavelet functions allow the cell sizes used in the MRTD technique to approach the Nyquist limit ($\lambda/2$) [3]. The wavelet functions act as band-pass filters, complementing the low-pass filter action of the scaling function. Thus, the wavelet functions increase the frequency content modeled in the simulation. This property of wavelet analysis lays the foundation for the adaptive gridding technique.

MRTD algorithms have demonstrated unparalleled properties when applied to the analysis of structures with medium to large sized computational domains. Through a two-fold expansion of the fields in scaling and wavelet functions with respect to time/space, memory and execution time requirements are minimized while a high resolution in areas of strong field variations or field singularities is achieved through the use of sufficiently large number of wavelet resolutions. The major advantage of the MRTD algorithms is their capability to develop real-time time and space adaptive grids through the efficient thresholding of the wavelet coefficients.

Various expansion basis have been utilized for the implementation of the MRTD algorithms. The Battle-Lemarie basis offers a reduction in memory by 2-3 orders of magnitude for 3D structures. Nevertheless, the entire-domain character of these functions adds a significant computational overhead in the approximation of the field derivatives in Curl Maxwell equations. In addition, Hard Boundaries (e.g. PEC's) cannot be applied directly by zeroing out the appropriate field components; image theory has to be implemented to account for the neighboring cells' contribution. Due to their compact support, Haar expansion basis functions (Fig.1) provide schemes that are similar to the FDTD algorithm that can be derived using pulse basis. They do not provide the drastic economies of the entire-basis schemes, but can be implemented in a much simpler way and maintain the adaptive feature.

For simplicity, the 1D MRTD scheme for TEM propagation will be presented. It can be extended to 2D and 3D in a straightforward way. The Electric (E_x) and the Magnetic (H_y) fields are displaced by half step in both time- and space-domains (Yee cell formulation) and are expanded in a summation of scaling (ϕ) and wavelet (ψ) functions in space and scaling components in time. For example, E_x is given by

$E_x(z,t) = \Sigma_{m,i} \{{}_mE_{x,i}^{\phi}\ \phi_i(z) + \Sigma_{r=0->r\ max}\ \Sigma_{ir=1->2^r}\ {}_mE_{x,i}^{\psi\ r,ir}\ \psi_i^{r,ir}(z)\}\ \phi_m(t)$, where $\phi_i(z)=\phi(z\ \Delta z\text{-}i)$ and $\psi_i^{r,ir}(z)=2^{\ r/2}\ \psi_0(2^r\ (z\ \Delta z\text{-}ir)\text{-}i)$ represent the Haar scaling and r-resolution wavelet functions located inside the *i*-cell. The conventional notation ${}_mE_{x,i}$ is used for the Electric field component at time $t=m\ \Delta t$ and $z=i\ \Delta z$, where Δt and Δz are the time-step and the spatial cell size respectively. The notation for H_y is similar. Substituting E_x,H_y in the TEM equations and applying Galerkin technique derives MRTD equations. Dielectric interfaces are modeled through the discretization of the constitutive relationship $D=\varepsilon E$ and the solution of a matrix equation involving all scaling and wavelet components with domains containing the interface.

Due to the finite-domain nature of the expansion basis, the Hard Boundary conditions (Perfect Electric/Magnetic Conductor) can be easily modeled. For example, if a PEC exists at the $z=i\Delta z$, then the scaling E_x coefficient for the *i*-cell has to be set to zero for each time-step *m* since the position of the conductor coincides with the midpoint of the domain of the scaling function. Nevertheless, the 0-resolution wavelet for the same cell has the value of zero at its midpoint; thus its amplitude does not have to be set to zero. To enforce the physical condition that the electric field values on either side of the conductor are indpendent from the files on the other side, TWO 0-resolution wavelet E_x coefficients have to be defined. The one (on the one side of P.E.C.) will depend on H_y values on this side only and the other (on the other side of P.E.C.) will depend on H_y values on that side only. Wavelet coefficients of higher-resolution with domains tangential to the position of P.E.C. have to be zeroed out as well.It can be easily observed that for Wavelet Resolutions up to r_{max}, $2^{\ rmax+1}$ coefficients have to be calculated per cell per field component instead of one component in the conventional FDTD.

The fact that the wavelet coefficients take significant values only for a small number of cells that are close to abrupt discontinuities or contain fast field variations allows for the development of a dynamically adaptive gridding algorithm. One thresholding technique based on absolute and relative thresholds offers very significant economy in memory while maintaining the increased resolution in space where needed. For each time-step, the values of the scaling coefficients are first calculated for the whole grid. Then, wavelet coefficients with resolutions of increasing order are updated. As soon as all wavelet components of a specific resolution of a cell have values below the Absolute Threshold (that has to do with the numerical accuracy of the algorithm) or below a specific fraction (Relative Threshold) of the respective scaling coefficient, no higher wavelet resolutions are updated and the simulation moves to the update of the wavelet coefficients of the next cell. In this way, the execution time requirements are optimized, since for areas away from the excitation or discontinuities, only the scaling coefficients need to be updated. This is a fundamental difference with the conventional FDTD algorithms that cannot provide a dynamical time- and space-adaptivity even with grids of variable cell sizes (static adaptivity).

MEMS CAPACITOR MOTION MODELING

On-chip capacitors are valuable matching and tuning components in most RF circuits. MEMS capacitors have demonstrated very low loss and significantly higher Q in comparison to conventional ones, without increasing space requirements. Modeling of this type of structure is challenging since a model must incorporate the motion of the plates under the combined effect of a DC bias and an RF excitation. The capacitor presented herein is a parallel plate capacitor that exhibits one-dimensional motion [4].

The modeled capacitor is presented in Figure 1. It is comprised of two plates, the bottom fixed, the top restrained by a spring and damper. The spring represents the force from the support of the top plate, while the damper represents air resistance. With no applied bias the weight and spring force on the top plate reach equilibrium (the damper only has an effect when the plate is in motion). When a bias is applied, the electrostatic force on the top plate is represented by

$$f = \frac{\varepsilon_\circ A V^2}{(x-h)^2} \qquad (1)$$

In this equation, A is the area of the plate, V is the voltage between the plates, h is the initial separation of the plates, and x is the displacement of the top plate from its initial position. This equation neglects fringing fields around the capacitor, and is accurate when the plate size is large compared to the separation. In addition, it is noted that while the voltage is constant, the force changes based on the position of the top plate. In an RF circuit, V varies due to the propagating RF field.

The equation of motion of the top plate is the standard 2nd order ordinary differential equation for spring mass systems with (1) as a forcing function. The equation [4] is

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = \frac{\varepsilon_\circ A V^2}{(x-h)^2} \qquad (2)$$

b is the damping coefficient, k is the spring coefficient, and m is the mass of the plate. The capacitor undergoes damped oscillatory motion.

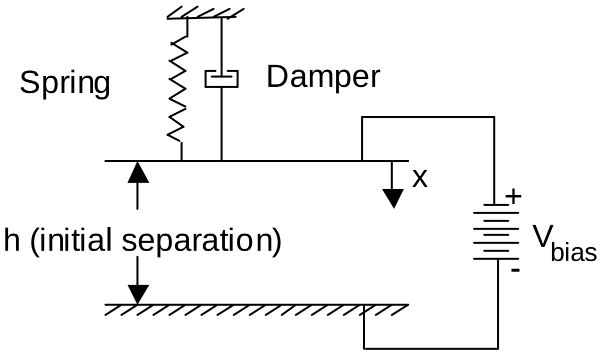

Figure 1: Schematic of Capacitor

The purpose of this investigation is to combine the mechanical motion of the parallel plate capacitor with a time domain electromagnetic simulator. As such, it is necessary to have a time domain simulation of the capacitor's motion. A finite difference discretization of the above model is appropriate. In order to create a model with second order accuracy, central differences are employed.

Using the standard notation

$$u(n\Delta t) = u^n \tag{3}$$

(2) can be written as:

$$m\frac{x^{n+1} - 2x^n + x^{n-1}}{\Delta t^2} = -b\frac{x^{n+1} - x^{n-1}}{2\Delta t} - kx^i + \frac{\varepsilon_\circ AV^2}{(x^n - h)^2} \tag{4}$$

Solving (4) for x^{n+1} gives

$$x^{n+1} = 2x^n\left(\frac{2m - k\Delta t^2}{2m + b\Delta t}\right) + x^{n-1}\left(\frac{b\Delta t - 2m}{b\Delta t + 2m}\right) + \frac{2\Delta t^2 \varepsilon_\circ AV^2}{(2m + b\Delta t)(x^n - h)^2} \tag{5}$$

When implemented, it is assumed that x is equal to zero for all time prior to t=0. This equation could be used by itself to determine the motion of the parallel plate under a fixed bias voltage V. However, it should be noted that V could be varied at any time, and the equation would be able to react to the change. This is very important when combining the equation with an electromagnetic simulator.

APPLYING MOTION TO AN EM MODEL

In order to efficiently simulate the motion of the MEMS capacitor under an RF excitation, the above model must be integrated into an electromagnetic simulator. Two types of full wave time-domain simulators will be discussed, the Finite-Difference Time-Domain (FDTD) and Multiresolution Time-Domain (MRTD) techniques. There are several similarities between these two modeling methods, and some important differences that affect the implementation.

In either type of simulator the plates are represented as metals, the bottom one fixed. The position of the top plate, however, changes with time. Because both simulators are time domain, a change in the boundary conditions, such as would be caused by a moving metal, is not necessarily a problem. Indeed, because the boundary conditions are enforced explicitly at every time step, a moving plate simply causes the boundary conditions to be enforced at different space points for each time step. In this manner, a time varying metal plate is easy to incorporate into an FDTD or MRTD model. In addition, the voltage between the plates caused by the interaction of the capacitor with an applied RF field can readily be calculated from the EM simulator. Thus the bias voltage and voltage due to the applied field can be combined to correctly calculate the forcing function. There are, however, several characteristics of the capacitor geometry that make FDTD modeling difficult.

As stated previously, the separation between the plates is very small compared to their width. In order to accurately simulate the capacitor it is important to have several cells between the plates. This creates a very small cell side length compared to the plate width and the dimensions of any feeding structure. While all three sides of the cell are not required to be the same length, a large aspect ratio between cell side lengths causes numerical inaccuracies. In order to maintain a reasonable ratio between the cell side lengths, the cells used in the simulation must be made very small compared to the computational space. However, the large number of grid points this causes in the simulation are computationally prohibitive. This discretization problem is linked to the dominant difficulty of MEMS modeling, coupling the position results provided by a mechanical simulation to a space-dependent electromagnetic simulation.

The equation of motion (5) is an ordinary differential equation. As such, it is discretized only in time, not space and time. The spatial variable can take on any value. This creates a problem in its integration into the electromagnetic equations, which are discretized in both space and time (as in FDTD and MRTD). In order to simulate the moving metal plate in a fixed spatial grid, the plate must take on one of a discrete number of spatial points. There are at least two ways to handle this difficulty in FDTD. The first method is to find the spatial position of the plate from the motion equation, and apply the plate boundary conditions at the closest grid points. When the motion equation is next updated, the exact computed spatial value would be used for the update. Thus, the electromagnetic simulator would use the averaged values while the motion simulator would not. This would introduce error in several ways.

The first error caused by this method is that the spatial position of the capacitor plate is not exact. The equation for the capacitance of a parallel plate capacitor is

$$C = \frac{\varepsilon_\circ A}{d} \tag{6}$$

If d is not represented by a large number of cell widths, the capacitance using the discretized grid points may be unacceptably different from the exact capacitance. These errors would compound during the execution of the problem.

Another option for simulating the capacitor would be to modify the FDTD grid at each time step. The new grid would have a grid level at the exact height of the top plate. The inherent problem with this method is how to determine the field values at the new grid points. Obviously, some type of interpolation would have to be used. The error introduced by this changing grid would be difficult to determine. However, these problems can be alleviated using the adaptive grid provided by MRTD.

MRTD uses a wavelet field discretization. This allows cells to be significantly larger than in FDTD. The resolution of cells that contain fine geometry or high field variation can be increased by locally adding wavelets. These wavelets, which can be both time and space localized, are equivalent to an adaptive grid, which leads to reduced execution times (areas of small field variation are represented by large cell sizes) and improved memory efficiency. Thus, the MRTD technique can be used with the MEMS motion simulator to resolve the position of the top capacitor plate to any desired level of accuracy. As a rule of thumb, the position of the PEC is updated every 100-500 MRTD/FDTD time-steps and is modeled by zeroing out the appropriate scaling and wavelet coefficients. The maximum wavelet resolution can be determined by the local positioning error of the PEC on the MRTD cell.

TEST RESULTS

In order to test the ability of an FDTD simulator to adequately determine the capacitance of MEMS capacitor, a static MEMS capacitor was modeled. The capacitor used had an area of 450 μm^2, a plate separation of 1 μm, and was fed with a coplanar waveguide over a ground plane. The capacitor was treated as a one port structure, and Γ of the structure was determined. From Γ, the impedance and ultimately the capacitance were determined. The predicted capacitance, using (6), is 1.8 pF. Figure 2 shows a plot of capacitance vs. frequency. As can be seen, the capacitance rises slightly with frequency, and is lower than the predicted value. The change in capacitance with frequency, as well as the lower overall value, is due to parasitics from the feeding structure and the radiation from the capacitor due to fringing fields at the edges. The increasing trend agrees with previously published results [4].

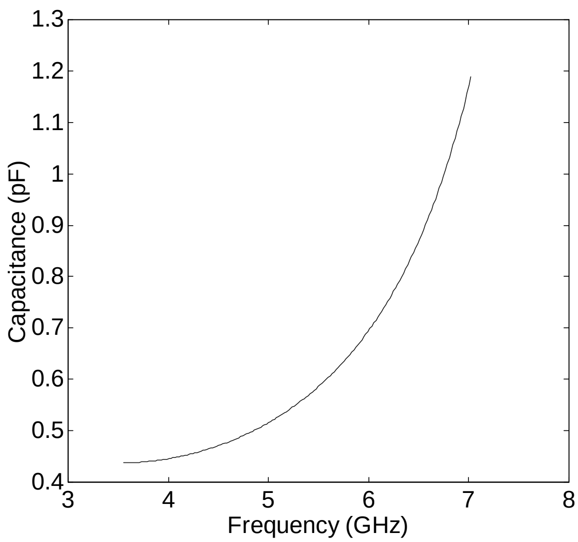

Figure 2: Capacitance of Test Device

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