Parametric Model Generation Algorithm for Planar Microwave Structures based on Full–Wave analysis and Design of Experiment

M. Al–Ahmad†, F. Coccetti†, M.Tentzeris‡ P. Russer†
†Institut für Hochfrequenztechnik, Technische Universität München
Arcisstrasse 21, D-80333 Munich, Germany, e-mail:russer@ei.tum.de
‡The School of Electrical and Computer Engineering Georgia Institute of Technology
Atlanta, GA 30332-0250,e-mail:etentze@ece.gatech.edu

Abstract — *The Design of Experiment approach (DoE) and an electromagnetic full–wave solver are combined in a single algorithm in order to produce parametric model of planar microwave and millimeter wave components. The DoE is a technique normally used to improve a process or a system and is here applied for conducting an electromagnetic analysis in which parameter levels, as geometrical dimensions, are made vary and the corresponding changes on the output are recorded. The results are then statistically processed by the Analysis of Variance (ANOVA) for determining which parameters or their combination is more significant. The electromagnetic analysis performed in time–domain by means of a Transmission Line Matrix (TLM) method provides a very broadband characterization. The given structure is simulated for each of the possible combination obtained from changing each parameter between two levels while keeping all the others constant. The two values correspond to the limit of a range of variation opportunely chosen. Once the parameters have been identified multiple linear regression (MLR) approach is applied to develop the model. This algorithm is here tested to two different test structures, as a microstrip (MS) patch antenna on teflon substrate and a coplanar waveguide (CPW) bend on silicon substrate. The accuracy analysis complete the algorithm description by giving an estimation of the model error .*

1 Introduction

Microwave and millimeter–wave components fabricated on sophisticated technology upon more and more demanding requirements of frequency bandwidth and functionality, has urged the need of techniques of analysis able to define reliable and accurate design rules suitable for the synthesis and the optimization of novel devices. Models for precise electromagnetic computer aid design (CAD) software are very often not existing or not accurate enough. Distributed RF circuits are fully characterized by their scattering matrix which are strictly dependent from the design parameters as geometrical dimensions and material properties. A first step in the definition of a parametric model therefore, consists in the selection of those factors which are more significant and in the identification of their effects on the component performances. Among the existing prior approaches with the aim to derive analytic model of high frequency networks, there are look–up table, curve fitting techniques, space mapping (SM)[3], artificial neural networks (AAN) [4], multidimensional adaptive parameter sampling (MAPS)[5]. The concept which distinguishes the present method is the use of statistical tools as ANOVA, to discriminate which role and in which frequency range each of these parameter plays [6]. The block diagram for the proposed algorithm is depicted in Fig. 1. A structure under study depend on a certain number of parameters. Only part of these can vary independently while the rest depends from the first ones or are defined by prior knowledge and design criteria. A design of experiment is conducted in order to identify a number of electromagnetic simulations or experiments that is sufficient to provide objective statistical conclusions can be made. Different design strategies are available and depending on the number of parameters and the experiment's properties a full– or fractional–factorial analysis is possible. These two design strategies consist in analyzing all (full) possible parameter combinations between two possible values or taking only a portion (fractional) of these respectively[7]. The full-wave analysis is base on the time–domain TLM method, which offers beside great stability and accuracy, scattering parameters over a broad frequency band at the cost of a single simulation [2]. The data collected from the entire set of simulations are consequently processed with the ANOVA algorithm which yields a hierarchical list of the parameters and their combinations ordered in terms of their weight on the output performances (scattering parameters). The final model is eventually built by relating these parameters

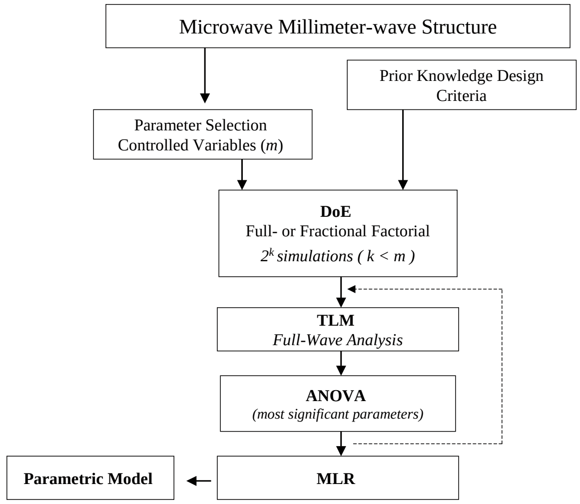


Figure 1: Block diagram of the Algorithm

and/or their combination through multilinear regression (MLR) technique. A broad band parametric model of the high frequency structure under study is therefore given available to further application in synthesis and optimization problems.

2 Theory

The DoE is a theory with the aim to define how to correctly set up an experiment (be it a measurements, a simulation or other) in order to achieve objective and valid conclusions. Among the existing DoE strategies for the multivariable esperiments the most complete is the full–factorial. The advantages of the full–factorial design is the efficiency and the possibility to detect variable interactions. On the other hand the number of runs (simulations) required can quickly become very large. For example for 6 variables and a two level full–factorial design a number of $2^6 = 64$ combinations are required. The possibility to reduce this number is offered by the fractional–factorial design, in which some combination are neglected because give no further analysis insight.

2.1 ANOVA Algorithm

ANOVA is a statistical technique able to quantify the significant differences between output levels, resulting in a hierarchy of parameters ordered for importance. An analysis of variance is brought out by comparing the variance within an output level against the variance across the whole level population. To apply the test a random sampling of a variable (simulation output) y with equal variance, independent errors, and a normal distribution are assumed. Let n be the number of replicates (set of identical observations) within each of K factor levels (treatment groups), and y_{ij} be the j^{th} observation within factor level i^{th}. ANOVA is "balanced" by restricting n to be the same for each factor level (each of the input columns is treated as a separate group, and each group has the same number of elements). A factor F is used to compare the sum of the squares of the levels with the sum of the squares of the entire data samples and is so defined:

$$F_{Statistic} = \frac{(SSE)(df2)}{(SSR)(df1)} \tag{1}$$

where the terms:

$$SSR = \frac{1}{n}\sum_{i=1}^{k}(\sum_{j=1}^{n}(y_{ij}))^2 - \frac{1}{Kn}(\sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij}))^2. \tag{2}$$

$$SSE = \sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij} - \bar{y_i})^2 = SST - SSR \tag{3}$$

with:

$$SST = \sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij} - \bar{\bar{y}})^2 \tag{4}$$

are the treatment sum of squared, the error sum of square, and the total sum of squared respectively. Here, $\bar{y_i}$ is the mean of observations within factor level i^{th}, and $\bar{\bar{y}}$ is the "group" mean (i.e., mean of means). The others two quantities, df_1 and df_2, are called column degree of freedom, calculated as the maximum element in a group minus one, and the error degree of freedom, given by the length of the input vector (here the structure response) minus the group's maximum respectively. The ANOVA F statistic factor can only assume non–negative values. It is zero if all the sample means are identical and gets larger as they get farther apart from each other[7][8].

The ANOVA analysis breaks down the generic electromagnetic response S in m frequency ranges in which the computed statistic factors indicate the dominant parameters and/or their combinations. In each i^{th} interval a parametric model $\phi_1(\theta_i, f)$ is build to reconstruct the response as shown in (5). The parameter vector θ_i has variable length corresponding to the number of selected parameters in the correspondent frequency interval. The resulting parametric model (MLR) can be written as follows:

$$S(f,\Theta) \quad = \begin{cases} \phi_1(\theta_1, f) & f_1 \leq f < f_2 \\ \phi_2(\theta_2, f) & f_2 \leq f < f_3 \\ \cdots & \\ \phi_m(\theta_m, f) & f_{m-1} \leq f < f_m \end{cases} \quad = \sum_{i=1}^{m} \phi_i(\theta_i, f) \sqcap_i (f) \tag{5}$$

where the function $\sqcap_i(f)$ is defined as:

$$\sqcap_i(f) = \begin{cases} 1 & f_{i-1} \leq f < f_i \\ 0 & elesewhere \end{cases} \tag{6}$$

2.2 MLR Model

To determine the relationship between regressor variables x and a response variable y for a designed experiment, the values of x, controlled by the experimenter, and the corresponding value y, experiment output, are recorded. Many regression problems involve more than one regressor variable. The generalized multiple linear regression model is given for k regressor variables by:

$$y = \underbrace{\beta_0 + \sum_{j=1}^{k}\beta_i x_i}_{Linear \quad Terms} + \underbrace{\sum\sum_{i<j}\beta_{ij}x_i x_j}_{Interactions \quad Terms} + \underbrace{\sum_{j=1}^{k}\beta_{jj}{x_j}^2}_{Quadratic \quad Terms} \tag{7}$$

This model in terms of the observations corrected to their averages (no constant term, β_0), may be written (if only linear terms are included), in matrix notation as:

$$\mathbf{Y} = \mathbf{X}\beta + \epsilon \tag{8}$$

where

$$\mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} (x_{11} - \bar{x_1}) & (x_{21} - \bar{x_2}) & \cdots & (x_{k1} - \bar{x_k}) \\ (x_{12} - \bar{x_1}) & (x_{22} - \bar{x_2}) & \cdots & (x_{k2} - \bar{x_k}) \\ \vdots & \vdots & \ddots & \vdots \\ (x_{1n} - \bar{x_1}) & (x_{2n} - \bar{x_2}) & \cdots & (x_{kn} - \bar{x_k}) \end{bmatrix}$$

and

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix} \quad \epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}.$$

The j^{th} column of the data matrix $\mathbf{X}$ might represent linear interactions or nonlinear terms, $\bar{x}_j$ is the average of the j^{th} column. In general, $\mathbf{Y}$ is an $n \times 1$ vector of responses, $\mathbf{X}$ is an $n \times k$ matrix of the levels of the regressor variables, β is a $k \times 1$ vector of frequency dependent regression coefficients, and ϵ is a random error vector $n \times 1$. The dimension "k" has to be replaced by a larger number if cross–variable effects get included in the regression process [9]. The target is to find the vector of least squares estimators $\hat{\beta}$ that minimize

$$L = \sum_{j=1}^{n} \epsilon^2 = \epsilon' \epsilon = (\mathbf{Y} - \mathbf{X}\beta)'(\mathbf{Y} - \mathbf{X}\beta) \tag{9}$$

The least squares estimators $\hat{\beta}$ must satisfy

$$\frac{\partial L}{\partial \beta}|_{\hat{\beta}} = -2\mathbf{X}'\mathbf{Y} + 2\mathbf{X}'\mathbf{X}\hat{\beta} = 0 \tag{10}$$

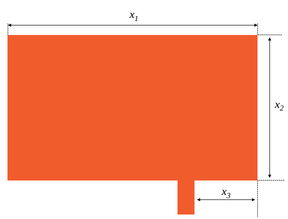
Solve for the least squares estimators of β

$$\hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} \tag{11}$$

The algorithm just mentioned is applied to fit the parameters vector θ_i to the scattering parameters (y) through the calculated set of regression coefficients $\hat{\beta}$.

3 Application Examples

The first example for which the above described algorithm has been applied is a misrostrip rectangular patch antenna on teflon substrate ($\varepsilon_r = 2.2$) feed by a feeding line of 50Ω. The three chosen parameters are described in Fig.2 with their correspondent 2–levels range of variability. A full-factorial design has been



Parameter	Value Range [min, max] in mm
x_1	$12.5 \div 23.5$
x_2	$16.0 \div 30.0$
x_3	$2.0 \div 4.0$

Figure 2: Patch antenna (top-view) and parameters variation range

applied with an overall amount of $2^3 = 8$ TLM simulations. Then the ANOVA analysis and the model construction by MLR a test structure has been used to validate the model accuracy by comparing the model results with those from the TLM (Fig.3). The second example is a coplanar waveguide bend, on silicon substrate ($\varepsilon_r = 11.7$) realized in copper ($\sigma = 5.8\ 10^7 S/m$) with compensation by means of airbridges. The structure is depicted in Fig. 4 together with the 4 parameters and the correspondent 2–levels range. A full-factorial design has been carried out, with 16 full-wave simulations. The results of the ANOVA analysis highlights that the most significant parameter is the width of the inner conductor and then the distance signal–to–ground, while the bridge position and width for the given range seems to give some contribution only above 55 GHz. The results comparison between model MLR and the correspondent TLM simulation are depicted in Fig. 5. In this synthesis problem the goal has been to bring the value for the transmission loss below $-2dB$ at 85GHz obtaining a set of parameters $[x_1, x_2, x_3, x_4] = [27, 115, 112, 35]$. The results in terms of scattering parameters are -17 dB for the return loss and -2.25 dB for the transmission loss at 85 GHz.

4 Error Analysis

Typically, the estimated coefficient $\hat{\beta}$ will follow a normal distribution due to the effect of the Central Limit Theorem. The distribution of the error is symmetric about its mean, usually around the mid points of the regression intervals and has a standard deviation σ_{error}, which has a value that practically all of the distribution (99.73%) lies inside the range $\mu - 3\sigma_{error} \leq x \leq \mu + 3\sigma_{error}$, for a standard normal distribution with $(\mu, \sigma^2) = (0, 1)$. The use of non–linear regression techniques can improve the algorithm accuracy by reducing the model error.

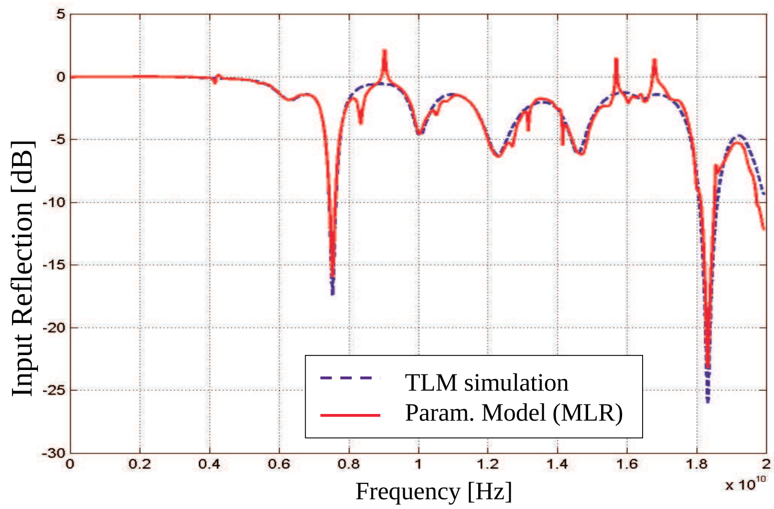
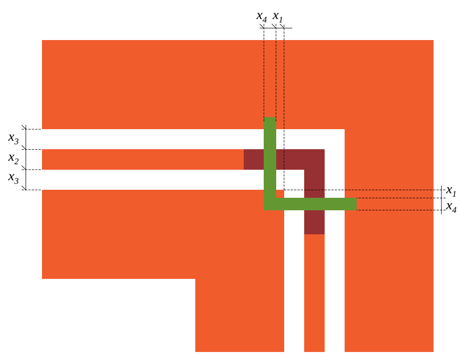


Figure 3: Comparison results for a synthesized patch antenna input reflection obtained from the model and from the TLM simulation



Parameter	Range [min, max] in μm	Description
x_1	$0 \div 100$	Air Bridge Position
x_2	$100 \div 200$	CPW signal line width
x_3	$100 \div 150$	CPW slot width
x_4	$30 \div 50$	air bridge width

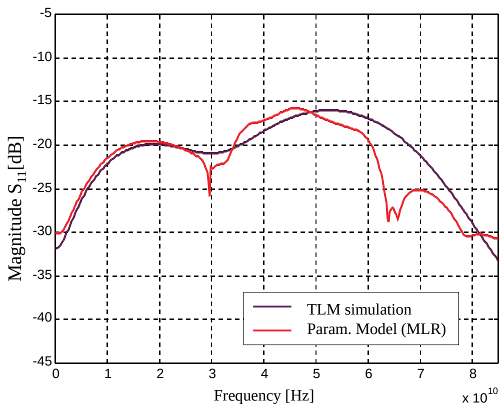
Figure 4: CPW bend (top-view) with the chosen parameters and the correspondent variation range

5 Conclusions

An algorithm based on design of experiment approach and full–wave simulations has been presented. The ANOVA technique applied to the set of simulation results enables the selection of the most significant parameters among a total number which can be even very large. The stability and elasticity of the TLM solver allows the electromagnetic analysis of any structure of high complexity, with the advantage of a broad band characterization. The final parametric model is carried out with the MLR technique, and can replace the full–wave model in synthesis problems with an enormous saving of computational time. The presented algorithm has been applied to two benchmark structures, as a microstrip patch antenna and the other and a bend on a coplanar waveguide. A full–wave simulation of test structures has eventually been compared with the model. The algorithm can be extended to any electromagnetic planar structure and constitutes a base for a general technique of component library generation.

6 Acknowledgments

The authors are indebted to the Deutsche Forschungsgemeinschaft for supporting the stay of Prof. M. Tentzeris at the Technische Universität München. A deep gratitude is moreover addressed to the Evangelisch Hochschulgemeinde for contributing to the Mr. Al–Ahmad scholarship.



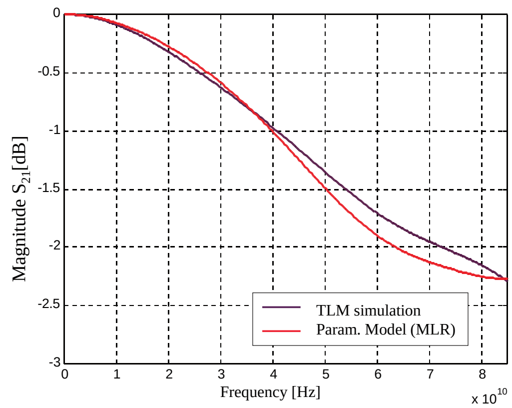


Figure 5: Comparison results for a CPW bend parametric model and its correspondent full–wave analysis by TLM in terms of scattering parameters magnitude

References

[1] P.B. Johns, R.L. Beurle, "Numerical Solution of 2-Dimensional Scattering Problems using a Transmission-Line Matrix," *Proc. IEE*, Vol.118, No. 9, pp. 1203-1208, Sept. 1971.

[2] L.Pierantoni, M.Farina, T.Rozzi, F.Coccetti, W.Dressel, and P.Russer, "Comparison of the Efficiency of Electromagnetic Solvers in the Time– and Frequency–Domain for the accurate Modeling of Planar Circuits and MEMS," *IEEE MTT–S Int. Symp. Dig.,*Seattle, USA, June 2002

[3] J.W.Bandler, R.M.Biernacki, S.H.Chen, P.A.Grobelny and R.H.Hemmers,"Space Mapping Technique for Electromagnetic Optimization," *IEEE Trans. Microwave Theory and Tech.,*Vol.42, No. 12, pp. 2536–2544, Dec. 1994.

[4] F.Wang, V.K.Devabhaktuni and Q.J.Zhang,"A Hierachical Neural Network Approach to the Development of Library of Neural Models for Microwave Design,"*IEEE MTT–S Int. Symp. Dig.,*Baltimora, USA, June 1998, pp. 1767–1770.

[5] J.De Geest, T.Dhaene, N.Fach' and D.De Zutter"Adaptive CAD–Model Building ALgorithm for General Planar Microwave Structures," *IEEE Trans. Microwave Theory and Tech.,*Vol.47, No. 9, pp. 1801-1809, Sept. 1999.

[6] D.Staiculescu, J.Laskar and M.Tentzeris, "Design Rule Development for Microwave Flip–Chip Applications," *IEEE Trans. Microwave Theory and Tech.,*Vol.48, No. 9, pp. 1476–1481, Sept. 2000.

[7] Douglas C.Montgomery, "Design and Analysis of Experiments," New york – John Wiley , 3^{rd} Edition 1991.

[8] Irving W.Burr, "Applied Statistical Methods," New york – Academic press, 1974.

[9] N. Draper and H. smith, "Applied Regression Analysis," New york – John Wiley , 3^{rd} Edition 1998.