Time Adaptive Time-Domain Techniques for the Design of Microwave Circuits

Manos M. Tentzeris, James Harvey, and Linda P. B. Katehi

***Abstract*—A novel time adaptive time-domain technique based on the Haar expansion basis is proposed and validated for a specific circuit problem. The modeling of active and passive lumped and distributed elements, as well as of excitation and boundary conditions, is performed effectively. This scheme, based on a combination of absolute and relative thresholding, provides a real-time time adaptive grid with improved time resolution in comparison to conventional time-domain schemes (FDTD) while maintaining a similar accuracy.**

***Index Terms*— Adaptive gridding, memory compression, microwave circuits, multiresolution, thresholding, time-domain techniques, wavelets.**

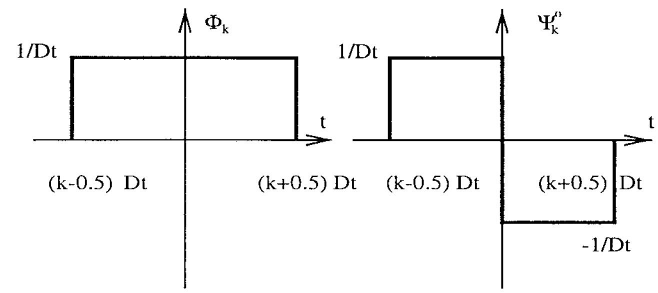

Fig. 1. Zero-order intervalic function basis.

I. Discussion on the HAAR Expansion Basis

SIGNIFICANT attention is being devoted now-a-days to the analysis and design of various types of microwave circuits. The finite-difference-time-domain (FDTD) scheme is one of the most powerful numerical techniques used for numerical simulations. However, despite its simplicity and modeling versatility, the FDTD scheme suffers from serious limitations due to the substantial computer resources required to model electromagnetic problems with medium or large computational volumes. The multiresolution time-domain method (MRTD) [1]–[4] has shown unparalleled properties in comparison to Yee's FDTD. In an MRTD scheme the fields are represented by a twofold expansion in scaling and wavelet functions with respect to time–space. Scaling functions guarantee a correct modeling of smoothly varying fields. In regions characterized by strong field variations or field singularities, higher resolution is enhanced by incorporating wavelets in the field expansions. The major advantage of the use of Multiresolution analysis to time domain is the capability to develop time and space adaptive grids.

MRTD schemes based on cubic spline Battle–Lemarie scaling and wavelet functions have been successfully applied to the simulation of two-dimensional (2-D) and three-dimensional (3-D) open and shielded problems [1]–[3]. The functions of this family do not have compact support, thus the MRTD schemes have to be truncated with respect to space. Nevertheless, dispersion analysis of this MRTD scheme shows the capability of excellent accuracy with up to almost 2 points/wavelength (Nyquist Limit). However, specific circuit problems may require the use of functions with compact support. Especially in the approximation of time derivatives, the use of entire domain expansion basis would require very high memory resources for the storage of the field values everywhere on the grid for the whole or a large fraction of the simulation time. This problem does not exist in the approximation of the spatial derivatives since the field values on the neighboring spatial grid points have to calculated and stored no matter what expansion basis are used. For that reason, Haar basis functions have been utilized in space-domain [6]. As an extension to this approach, intervalic wavelets (Fig. 1) may be incorporated into the solution of SPICE-type circuits in time domain. Results from that new technique will be presented in this letter.

II. Applications in SPICE Problems

For simplicity, the one-dimensional (1-D) MRTD scheme will be derived. It can be extended to 2-D and 3-D in a straightforward way. In addition, only the 0-resolution of wavelets is enhanced. The voltage and the current are displaced by half step in both time- and space-domains (Yee cell formulation) and are expanded in a summation of scaling functions in space and scaling (ϕ) and wavelet (ψ_0) components in time. For example, the voltage is given by

$$V(z, t) = \sum_{m=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} ({}_iV_m^{\phi}\phi_i(t) + {}_iV_m^{\psi_0}\psi_{0,i}(t))\phi_m(z) \tag{1}$$

where $\phi_i(t) = \phi(t/\Delta t - i)$ and $\psi_{0,i}(t) = \psi_0(t/\Delta t - i)$ represent the 0-order intervalic scaling and 0-resolution wavelet functions. The conventional notation ${}_kV_m$ is used for the voltage component at time $t = k\Delta t$ and $z = m\Delta z$, where Δt and Δz are the time-step and the spatial cell size, respectively. The notation for the current I is similar.

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Due to the finite-domain nature of the expansion basis, the hard boundary conditions (open/short circuit) can be easily modeled. If a short circuit exists at the $z = m\Delta z$, then both scaling and wavelet voltage coefficients for the $m-$cell must be set to zero for each time-step k (${}_kV_m^{\phi} = {}_kV_m^{\psi_0} = 0, k = 0, 1, \cdots$). Similarly, an open circuit at $z = (m - 0.5)\Delta z$ can be modeled by zeroing out the current coefficients (${}_{k-0.5}I_{m-0.5}^{\phi} = {}_{k-0.5}I_{m-0.5}^{\psi_0} = 0, k = 0, 1, \cdots$).

The alternating nature of the 0-resolution wavelet function guarantees the double time-domain resolution of the MRTD scheme. Updating the voltage scaling and wavelet coefficients at $m = \Delta z$ for a specific time-step k, two values can be defined for the time span $[(k - 0.5)\Delta t, (k + 0.5)\Delta t]$ of this time-step

$$ {}_kV_m^{\text{total}} = \begin{cases} {}_kV_m^{\phi} + {}_kV_m^{\psi_0}, & t \in [(k-0.5)\Delta t, k\Delta t] \\ {}_kV_m^{\phi} - {}_kV_m^{\psi_0}, & t \in [k\Delta t, (k+0.5)\Delta t]. \end{cases} \quad (2) $$

Without loss of generality, the derivation of the MRTD equations for a lossless transmission line will be presented. The time-domain voltage variation in this type of transmission lines is described by

$$ \frac{dI}{dz} = -G_{\text{dis}}V - C_{\text{dis}}\frac{dV}{dt}. \quad (3) $$

Following a procedure similar to the [2], the following MRTD equations are derived

$$ \begin{aligned} {}_{k+1}V_m^{\phi} = &-\frac{(C_1 - C_2)}{C_1^2}\Delta t \Delta I^{\phi} + \frac{C_2}{C_1^2}\Delta t \Delta I^{\psi 0} \\ &+ \frac{(C_1 - C_2)^2 + C_2^2}{C_1^2}\, {}_kV_m^{\phi} - 2\frac{C_2^2}{C_1^2}\, {}_kV_m^{\psi_0} \end{aligned} \quad (4) $$

$$ \begin{aligned} {}_{k+1}V_m^{\psi_0} = &-\frac{C_2}{C_1^2}\Delta t\, \Delta I^{\psi} - \frac{(C_1 + C_2)}{C_1^2}\Delta t\, \Delta I^{\psi 0} \\ &- 2\frac{C_2^2}{C_1^2}{}_kV_m^{\phi} + \frac{(C_1 + C_2)^2 + C_2^2}{C_1^2}{}_kV_m^{\psi_0} \end{aligned} \quad (5) $$

with $\Delta I^{\phi} = {}_{k+0.5}I_{m+0.5}^{\phi} - {}_{k+0.5}I_{m-0.5}^{\psi}$, $\Delta I^{\psi 0} = {}_{k+0.5} I_{m+0.5}^{\psi 0} - {}_{k+0.5} I_{m-0.5}^{\psi 0}$, $C_1 = C_{\text{dis}}\Delta z$, $C_2 = 0.5 G_{\text{dis}}\Delta z \Delta t$. The equations for the time-domain variation of the current scaling and wavelet coefficients are derived in a similar way after making use of the duality principle ($C_{\text{dis}} \rightarrow L_{\text{dis}}$, $G_{\text{dis}} \rightarrow R_{\text{dis}}$). The $L_{\text{dis}}, C_{\text{dis}}$ are the distributed inductance and capacitance of the line and $R_{\text{dis}}, G_{\text{dis}}$ are the conductor and dielectric loss, respectively.

For nonzero loss coefficients, the equations giving the scaling and wavelet coefficients for voltage and current are coupled. For lossless lines, (4) updating the scaling coefficients only get uncoupled of (5) updating the wavelet coefficients. To create an efficient time adaptive algorithm, all four equations must be coupled on any case. An efficient way is to apply the excitation in a physically correct manner. If the excitation has the time-dependence $g(t)$ at the location $z = m_e\Delta z$, then the scaling and wavelet coefficients for this cell have to be

$$ \begin{Bmatrix} {}_kV_{m_e}^{\phi} \\ {}_kV_{m_e}^{\psi_0} \end{Bmatrix} = \int_{(k-0.5)\Delta t}^{(k+0.5)\Delta t} g(t) \begin{Bmatrix} \phi_k(t) \\ \psi_{0,k}(t) \end{Bmatrix} dt. \quad (6) $$

Nevertheless, (6) has to be applied in order to satisfy the physical boundary condition at the excitation cell(s). It has to be noted, that (4) and (5) can be used only for lossy lines with low to medium loss coefficients. The threshold $C_2 \leq 5 \times 10^{-4} C_1$ for G_{dis} gave satisfactory results for all simulations. For higher loss coefficients, the loss can be modeled in an exponential way similar to [3]. For example, for large values of G_{dis} ($C_2 > 5 \times 10^{-4} C_1$), (4) and (5) have to be replaced by the following uncoupled expressions with $C_5 = G_{\text{dis}}\Delta t / C_{\text{dis}}$:

$$ \begin{Bmatrix} {}_{k+1}V_m^{\phi} \\ {}_{k+1}V_m^{\psi_0} \end{Bmatrix} = e^{-C_5} \begin{Bmatrix} {}_kV_m^{\phi} \\ {}_kV_m^{\psi_0} \end{Bmatrix} - e^{-0.5C_5}\frac{\Delta t}{C_1} \begin{Bmatrix} \Delta I^{\phi} \\ \Delta I^{\psi_0} \end{Bmatrix} $$

Using this procedure, a termination layer similar to the FDTD widely used perfectly matched layer (PML) [5] can be easily modeled. The $R_{\text{dis}}, G_{\text{dis}}$ should have a spatial parabolic distribution with very high maximum value and they should satisfy the condition $R_{\text{dis}} = G_{\text{dis}}C_{\text{dis}}/L_{\text{dis}}$ for each cell of the layer. In this way, one matched transmission line can be simulated by choosing the appropriate $R_{\text{dis}}, G_{\text{dis}}$ that satisfy the specified numerical reflection (usually < -80 dB).

Lumped passive elements such as capacitors, inductors, and resistors can be modeled in a similar way with the Distributed ones by numerically distributing them along one cell. For example, if one lumped capacitor C_{lum} is located at $z = m\Delta z$ along a lossy line with ($R_{\text{dis}}, G_{\text{dis}}, L_{\text{dis}}, C_{\text{dis}}$), the voltage coefficients ${}_{k+1}V_m^{\phi}$, ${}_{k+1}V_m^{\psi_0}$ will still be given by (4) and (5). The only difference is that the constant C_1 will have the new value $C_1 = C_{\text{dist}}\Delta z + C_{\text{lum}}$.

III. VALIDATION—THRESHOLDING ALGORITHM

To validate the above approach, the MRTD algorithm was applied to the simulation of a lossy transmission line with $(L_{\text{dis}}, C_{\text{dis}}, R_{\text{dis}}, G_{\text{dis}}) = (20 \text{ nH/m}, 3 \text{ nF/m}, 0.1 \ \Omega/\text{m}, 10^{-5} ℧/\text{m})$ for a Gabor excitation [50, 100 MHz], 5000 time-steps with size $\Delta t = 0.79\Delta t_{\max}$ and 4000 cells with $\Delta z = 15$ cm. Fig. 2(a), which displays the voltage scaling and wavelet coefficients evolution at $z = 1500\Delta z$ for the first 1400 time-steps of the simulation, shows that the wavelet coefficients have significant values only at areas with significant scaling function values and approximately equal to the values of the first derivative of the voltage spatial distribution and are close to the 11% of the respective scaling functions. Fig. 2(b) and (c) compares the total voltage value at the same probe position calculated by FDTD (Scal.) and MRTD (Scal.+Wav0) for the time-steps 1000–1400 and 1103–1107, respectively, and demonstrate the ability of this MRTD scheme to double the conventional FDTD resolution in the time domain by providing two values for each time-step.

The fact that the wavelet coefficients take significant values only for a small number of time-steps allows for the development of a time adaptive gridding algorithm. One thresholding technique based on absolute and relative thresholds offers very significant economy in memory while maintaining the double resolution in time where needed. For each time-step the maximum value of the voltage scaling coefficient over the whole grid is identified. All wavelet components with values below a specific fraction (relative threshold) of the above number are eliminated. To take into consideration the time-steps that the voltage scaling components have a very small value (close to numerical accuracy of the algorithm),

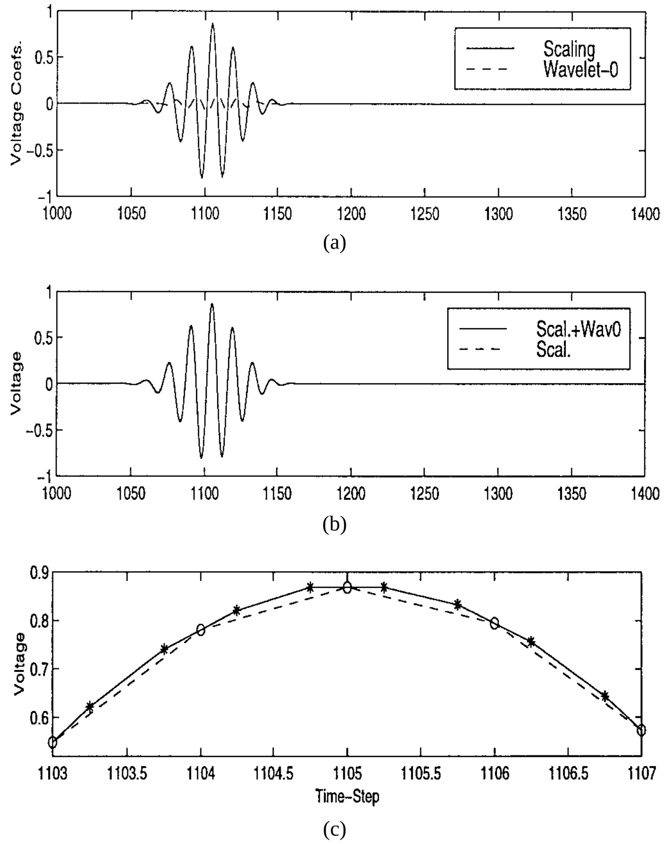

Fig. 2. Demonstration of the double resolution in time domain.

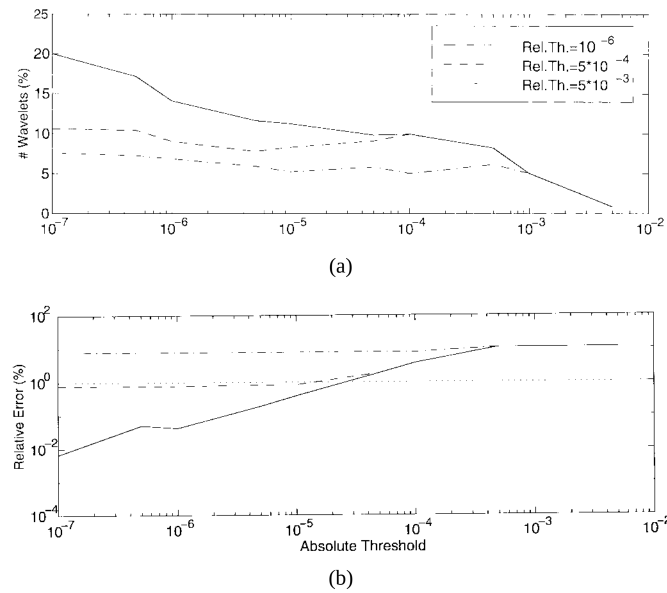

Fig. 3. Parametric investigation of absolute and relative threshold values.

an absolute threshold is introduced. Similar approach is used for the current wavelet coefficients. It has to be noted that the absolute threshold for the current components is a scaled version of the absolute threshold for the voltage components in order to account for their time-lag by half time-step as well as for their relationship through the wave impedance of the respective medium(s). Different values of the relative and absolute thresholds are investigated in Fig. 3 in terms of memory compression (M.C) and relative error (R.E.)

$$\text{M.C. } (\%) = \frac{\text{Number of Wavelets above thresholds}}{\text{Total number of Wavelets (=Grid Size)}} \times 100\% \tag{7}$$

$$\text{R.E. } (\%) = \frac{||\text{Voltage}_{(Th)} - \text{Voltage}_{(NTh)}||_2}{||\text{Voltage}_{(Nth)}||_2} \times 100\% \tag{8}$$

where $\text{Voltage}_{(Th)}$ and $\text{Voltage}_{(NTh)}$ are the total voltage values at the probe position at $z = 1500\Delta z$ with and without the use of the thresholding algorithm and $||x||_2 = \sqrt{\sum_{t=0}^{N_t} |x_t|^2}$ is the Norm-2 defined over the total number N_t of the simulation time-steps.

Fig. 3 displays the performance for absolute thresholds (T_{ab}) between 10^{-7} and 5×10^{-3} and relative thresholds (T_{re}) of 10^{-6}, 5×10^{-4}, and 5×10^{-3}. For values of T_{ab} above 10^{-3}, the M.C. falls below 5% (almost no wavelets are used) and the additional accuracy offered by the wavelets is lost (R.E. $\approx$ 10 %), independently of the value of T_{re}. On the contrary, for lower values of T_{ab}, the effect of T_{re} is critical. For T_{ab} below 5×10^{-6}, small values of T_{re} (in the order of 10^{-6}) create very moderate M.C. (larger than 15%) and too good accuracy (better than 0.05%), though large values of T_{re} (in the order of 5×10^{-3}) cause the opposite effect (M.C. close to 7.5% and R.E, close to 10%). The best compromise between compression and accuracy can be achieved for T_{re} between 10^{-4} and 5×10^{-4} and T_{ab} between 10^{-6} and 10^{-5} (M.C. $\approx$10% and R.E. $\in$ [0.1%, 1%]), assuming that the amplitude of the excitation is 1. For this optimum thresholding choice, the proposed time adaptive technique offers a double resolution in time domain in comparison to the conventional FDTD scheme, while increasing the meory requirements only by $\approx$10%. This property is very important, especially for simulations of active devices, which would require a large number of time-steps with values significantly lower than the Courant limit in order to achieve the necessary time resolution if the conventional FDTD scheme was used.

IV. Conclusion

A novel time adaptive time-domain technique based on the Haar (intervalic) expansion basis is proposed and validated for a specific circuit problem. Active and passive lumped and distributed elements, as well as excitation and boundary conditions are modeled effectively. This real-time time-adaptive scheme, based on a combination of absolute and relative thresholding, exhibits significant improvement in time-domain resolution while maintaining a similar accuracy with the conventional FDTD technique.

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