Comments

Comments on "Numerical Errors in the Computation of Impedances by FDTD Method and Ways to Eliminate Them"

Nihad Dib, Michael Krumpholz, Emmanouil Tentzeris, and Linda P. B. Katehi

In the above paper,[1] a new formula for the calculation of the characteristic impedance of transmission lines, Z_0, using the FDTD method has been given. This formula eliminates the numerical errors due to the offset of voltages and currents in space and time. We have recalculated the two examples given in the above paper and confirmed the presented results. However, we would like to point out that using the exact formula for the characteristic impedance of a stripline [1] yields a value of $Z_0 = 50.2\ \Omega$ for the stripline investigated in the above paper. Thus, Fang's result for the real part of Z_0 (see Fig. 3(a) in the above paper) has an absolute error of about 8.6 Ω and a relative error of -17%. This is clearly unacceptable for practical use and one has to choose a much finer discretization to obtain reasonable results. Figs. 1 and 2 show our results for a discretization of $\Delta x = \Delta y = \Delta z = 0.125$ mm. The real and imaginary parts of the impedance are given in Ω. In this case, compared to the exact Z_0, the absolute and relative errors in the real part are around 1.2 Ω and -2.4%, respectively. It is clear from the figures that, for this example, Fang's new formula does not give any significant improvement over the other formulas for a discretization fine enough to obtain accurate results. Furthermore, note that in addition to the formulas cited in the above paper, we have also included the ratio $V_k(\omega)/I_{k-1}(\omega)$, which gives better results than the ratio $V_k(\omega)/I_k(\omega)$.

References

[1] B. Bhat and S. Koul, *Stripline-Like Transmission Lines for Microwave Integrated Circuits.* New York: Wiley, 1991, p. 66.

Authors' Reply by Jiayuan Fang and Danwei Xue

In our paper, a simple formula was provided to eliminate the numerical error caused by the offsets of voltages and currents in space and time. Of course, the smaller the space-step of the finite-difference grid is used, the smaller such an error will be. In response to Dib *et al.*'s comments, the variation of the numerical error with respect to the space-step dh is presented as follows.

Suppose the voltage v_k^n and the current i_k^n are offset by half a space-step as shown in Fig. 3, and half a time-step as well. Also

Manuscript received May 2, 1995.
The authors are with the Radiation Laboratory, Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI 48109-2122 USA.
IEEE Log Number 9414121.

[1] J. Fang and D. Xeu, *IEEE Microwave and Guided Wave Lett.*, vol. 5, no. 1, pp. 6–8, Jan. 1995.

Manuscript received June 8, 1995.
The authors are with the Department of Electrical Engineering, State University of New York at Binghamton, Binghamton, NY 13902-6000 USA.
IEEE Log Number 941422.

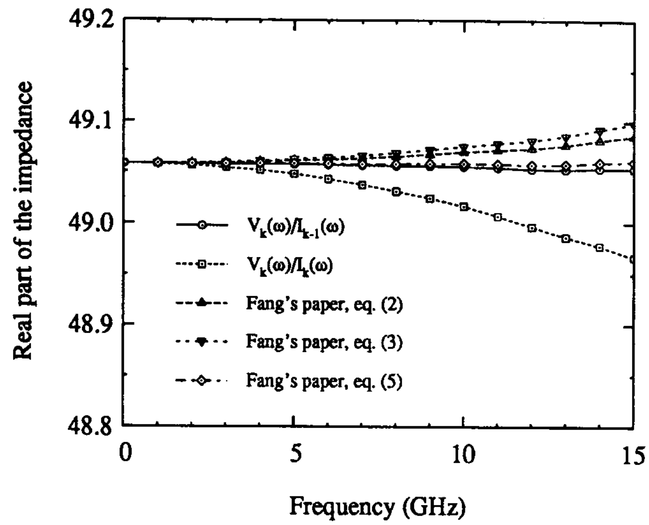

Fig. 1. Real part of the characteristic impedance of a stripline.

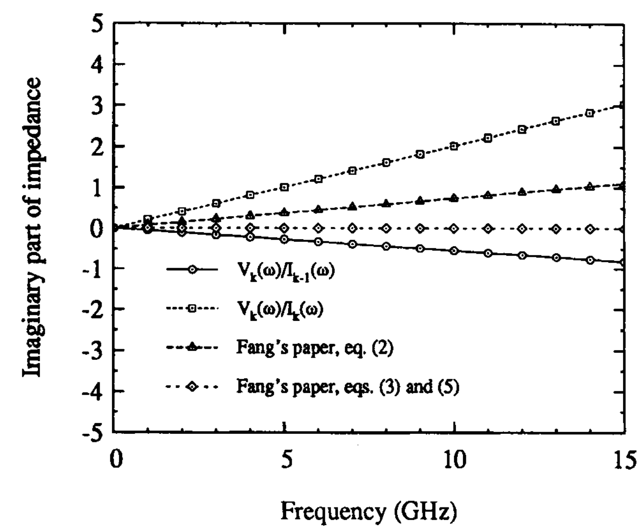

Fig. 2. Imaginary part of the characteristic impedance of a stripline.

suppose that the Fourier transforms of v_k^n and i_k^n are computed by

$$\begin{matrix} V_k(\omega) \\ I_k(\omega) \end{matrix} = \sum_n \begin{matrix} v_k^n \\ i_k^n \end{matrix} e^{-j\omega n\,\Delta t}\,\Delta t \tag{1}$$

where the half time-step offset between the voltage and the current is not accounted for. Assume the time-step Δt is chosen to be 0.5 dh/v in the following discussion, where v is the speed of light in the medium. If the current i_k^n leads the voltage v_k^n by half a time-step, then

$$V_k(\omega)/I_k(\omega) = Z_0(\omega)e^{j(k\,dh - \omega\,\Delta t)/2} \approx Z_0(\omega)e^{jk\,dh/4} \tag{2}$$

$$V_k(\omega)/I_{k-1}(\omega) = Z_0(\omega)e^{j(-k\,dh - \omega\,\Delta t)/2} \approx Z_0(\omega)e^{-j3k\,dh/4} \tag{3}$$

1051–8207/95$04.00 © 1995 IEEE

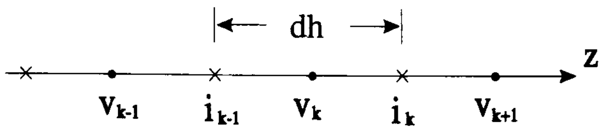

Fig. 3. Voltage and current positions.

where $Z_0(\omega)$ is the characteristic impedance without the error due to the spatial and time offsets of the voltage and the current. From (2) and (3), it can be seen that the error in the phase of $V_k(\omega)/I_k(\omega)$ is linearly proportional to the space-step dh, and is about one third of that in $V_k(\omega)/I_{k-1}(\omega)$. When $k\,dh$ is small, the error in the phase of the impedance will mainly appear as the error in the imaginary part of the impedance. On the other hand, if the current i_k^n lags the voltage v_k^n by half a time-step, then one can easily find that the error in the phase of $V_k(\omega)/I_k(\omega)$ is about three times of that in $V_k(\omega)/I_{k-1}(\omega)$.

Relative errors in the impedance calculated from (2) and (3) are

$$\left|\frac{\Delta Z_0}{Z_0}\right|_1 = |1 - e^{\pm j(k\,dh - \omega\,\Delta t)/2}| \tag{4}$$

$$\left|\frac{\Delta Z_0}{Z_0}\right|_2 = |1 - e^{\pm j(k\,dh + \omega\,\Delta t)/2}|. \tag{5}$$

Fig. 4 displays $|\Delta Z_0/Z_0|_1$ and $|\Delta Z_0/Z_0|_2$ versus λ/dh, where λ is the wavelength in the medium.

A general rule of thumb in selecting the space-step dh in FDTD computations is to ensure that the shortest wavelength λ of interest is about 10 ∽ 20 dh. As can be seen from Fig. 4, when $\lambda = 10\,dh$, $|\Delta Z_0/Z_0|_1$ and $|\Delta Z_0/Z_0|_2$ are about 16 and 47%; and when $\lambda = 20\,dh$, the errors expressed by $|\Delta Z_0/Z_0|_1$ and $|\Delta Z_0/Z_0|_2$ are about 8 and 24%. In our paper, $dh = 1$ mm is used, which corresponds to $\lambda = 10\,dh$ at the highest frequency of concern at 15 GHz. In the comments, the space-step dh is reduced to 0.125 mm, which corresponds to $\lambda = 80\,dh$ at $f = 15$ GHz. Relative errors in the impedances, computed from (4) and (5), are 2.0 and 5.9%, respectively, at $\lambda = 80\,dh$, which are consistent with those shown in Fig. 2. This level of error may still not be considered negligible even with such a fine grid.

As can be clearly seen from Fig. 4, the numerical error caused by just the space and time offsets in voltages and currents can be quite significant. This error mostly appears in the imaginary part and in the frequency-dependent behaviors of both the real and imaginary parts of the calculated impedance. Although one of the expressions in (2) and (3) gives smaller error than the other one, the simple formula provided in our paper completely eliminates such an error.

The other source of the numerical error in calculating the impedance mainly comes from the loss of field singularity at the edges of metal strips in the FDTD formulation. This error will

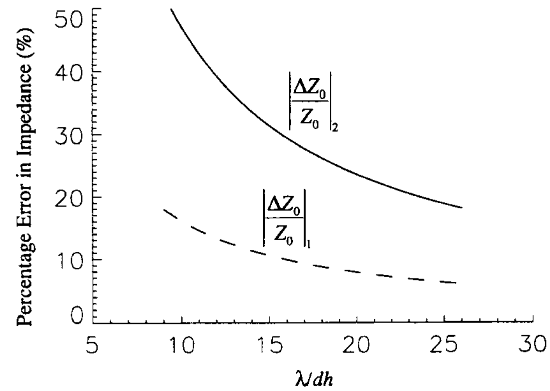

Fig. 4. Percentage errors in impedances calculated by $|V_k(\omega)/I_k(\omega)|$ and $|V_k(\omega)/I_{k-1}(\omega)|$.

mainly affect the real part of the calculated impedance. The formula given in our paper is not for the elimination of this type of error. Choosing an extremely fine grid is a simple but brute force approach. One of the alternative choices, with which one does not have to use a very fine grid to achieve significant effect in error reduction, would be to incorporate the field singularity behavior into finite-difference equations [1]. Another alternative is to make two computations of the impedance with two different space steps. Based on the fact that the error in the impedance is linearly proportional to the space step dh [2], a fairly accurate value of the impedance can be extrapolated. For the strip line example in our paper, the real part of the extrapolated impedance from those computed with $dh = 1$ mm (41.536 Ω) and $dh = 0.5$ mm (45.643 Ω) is 49.75 Ω, which is 0.45 Ω away from the exact solution of 50.2 Ω. The computation effort of two computations with $dh = 1$ mm and $dh = 0.5$ mm is much less than that of one computation with $dh = 0.125$ mm (very high degree of parallel computation may be an exception), but the impedance value thus obtained is much more accurate than that computed directly with $dh = 0.125$ mm (which has an error of 1.2 Ω). To reduce the error to 0.45 Ω with the direct fine grid computation, one has to use a space step less than 0.05 mm, which corresponds to $\lambda > 200\,dh$ at $f = 15$ GHz. With the techniques discussed above, together with the method in our paper, one does not have to choose a very fine grid to obtain accurate results.

References

[1] K. Beilenhoff and W. Heinrich, "Treatment of field singularities in the finite-difference approximation," in *1993 Int. Microwave Symp. Dig.*, 1993, vol. 2, pp. 979–982.

[2] P. Mezzanotte, L. Roselli, C. Huber, H. Zscheile, and W. Heinrich, "On the accuracy of the finite-difference method using mesh grading," in *1995 Int. Microwave Symp. Dig.*, 1995, vol. 2, pp. 781–784.